Calculus Section 9.7 Approximations and Lagrange Error Bound
-Use a Taylor or Maclaurin polynomial to approximate a function
-Find the Lagrange error for a polynomial approximation

Homework: Lagrange Error Bound Worksheet

One major application of Taylor and Maclaurin polynomials is approximating the value of a function.

**Example)**Use a 4th degree Taylor polynomial centered at 0 to approximate f(0.2) for the function f(x) = ex.

There is error associated with any approximation. We use the Alternating Series Remainder Theorem to find the maximum possible error for a convergent alternating series. But what if the series does not alternate?

**Lagrange Error Bound**Let Pn(x) be the nth degree Taylor polynomial for f(x) centered about x = c.
 $Error=\left|P\_{n}\left(x\_{0}\right)-f(x\_{0})\right|\leq \left|\frac{f^{\left(n+1\right)}\left(z\right)}{\left(n+1\right)!}(x\_{0}-c)^{n+1}\right|$

Where x0 is where the function is being approximated and $f^{\left(n+1\right)}\left(z\right)$ is the maximum value of the n + 1 term between x0 and c.

We do not need to find the value of z. We just need to find a reasonable, safe bound for $f^{\left(n+1\right)}\left(z\right)$.

**Example)**Use Lagrange error to prove that $\left|f\left(0.2\right)-P(0.2)\right|<\frac{1}{1000}$ for the previous example.

**Example)**
Let *f* be the function given by and let P(x) be the third-degree Taylor polynomial for *f* about x = 0. Find P(x). Then use the Lagrange error bound to show that.

**Example)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | f(x) | f ’(x) | f ’’(x) | f ’’’(x) |
| 1 | 10 | 5 | 6 | -2 |
| 2 | 6 | 4 | -7 | 8 |
| 3 | -3 | -2 | 9/4 | 3 |

Write a third-degree Taylor Polynomial for *f* about x = 2, and use it to approximate *f*(2.3). If the fourth derivative of *f* satisfies the inequalityfor all x in the interval, use the Lagrange error bound to find an interval [a,b] such that a ≤ *f*(2.3) ≤ b.