9.7 Lagrange Error Bound Worksheet

1) a) $P(x) = 1 + \frac{1}{2}x - \frac{1/4\, x^2}{2} + \frac{3/8\, x^3}{3!}$

b) $f(.5) \approx 1 + \frac{1}{2}(.5) - \frac{(.5)^2}{8} + \frac{3(.5)^3}{48}$

$f(.5) \approx 1.227$

c) $\frac{(\text{max})}{4!}(.5-0)^4$

$\frac{6}{4!}(.5)^4 \rightarrow \frac{1}{64}$ = Max Error

2) a) $f(x) = \sqrt{x}$ $\quad f(4) = 2$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ $\quad f'(4) = \frac{1}{4}$

$f''(x) = -\frac{1}{4}x^{-3/2} = \frac{-1}{4x^{3/2}}$ $\quad f''(4) = \frac{-1}{32}$

$P(x) = 2 + \frac{1}{4}(x-4) - \frac{1/32\,(x-4)^2}{2}$

b) $f(4.2) \approx 2 + \frac{1}{4}(4.2-4) - \frac{(4.2-4)^2}{64}$

$f(4.2) \approx 2.049$

c) $\frac{(\text{max})}{3!}(4.2-4)^3 = \frac{3/256}{6}(.2)^3 = 1.5625 \times 10^{-5}$ = Max Error

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad$ max occurs at $x=4$ $\rightarrow \frac{3}{8}(4)^{-5/2} = \frac{3}{8 \cdot 32} = \frac{3}{256}$

3) a) $P(x) = 1 + \frac{1}{2}(x-3) - \frac{1/4(x-3)^2}{2} + \frac{3/8(x-3)^3}{3!}$

b) $f(3.7) \approx 1 + \frac{1}{2}(3.7-3) + \frac{(3.7-3)^2}{8} + \frac{3(3.7-3)^3}{48}$

$f(3.7) \approx 1.310$

c) $\frac{(\text{max})}{4!}(3.7-3)^4$

$\frac{6}{4!}(.7)^4 = .060 < .08$

d) $1.310 - .060 = 1.25$

$1.310 + .060 = 1.37$

$1.25 \le f(3.7) \le 1.37$

yes, f(3.7) could equal 1.283

4) $\sin x = x - \frac{1}{6}x^3$

$\frac{(\text{max})}{4!}(.2)^4$ $\quad$ max of $|f''''(x)| = |-\cos(x)| = 1$ $\quad \frac{1}{4!}(.2)^4 = .000\overline{6}$ = Max Error

5) $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^{-x^2} = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{3!} + \frac{x^8}{4!}$

$\int_0^1 e^{-x^2}dx = \int_0^1 \left(1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24}\right)dx$

$\left[x - \frac{1}{3}x^3 + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216}\right]_0^1 - \frac{x^{11}}{1320}$ ← Alternating series Remainder

$\int_0^1 e^{-x^2}dx = \left(1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216}\right) - (0) \approx \boxed{.747}$

6) $\frac{(max)}{5!}(3-1)^5 \rightarrow \frac{.01}{5!}(2)^5 = \boxed{.002\overline{6}}$

7) $1 - \frac{1}{2!} + \frac{2}{3!} - \frac{3}{4!} + \frac{4}{5!} - \frac{5}{6!} \,\vdots\, + \frac{6}{7!}$ ← Alt. series Remainder $\boxed{.00119}$

8) $f^{(7)}(5) = \frac{(-1)^7\, 7!}{2^7(7+2)} = 4.375$

$\frac{(max)}{7!}(6-5)^7 \rightarrow \boxed{\frac{4.375}{7!} = .000868 < .001}$