9.8 Radius and Interval of Convergence II

Pg 654 #'s 45-47, 73-75

45) $f(x)=\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n$

$\lim_{n\to\infty}\left|\frac{x^{n+1}}{3^{n+1}}\cdot\frac{3}{x}\right|$

$\lim_{n\to\infty}\left|\frac{x}{3}\right|=\left|\frac{x}{3}\right|$

$\left|\frac{x}{3}\right|<1$

$-1<\frac{x}{3}<1$

$-3<x<3$

let $x=-3$

$\sum_{n=0}^{\infty}\left(\frac{-3}{3}\right)^n$

$\sum_{n=0}^{\infty}(-1)^n$

diverges by n^{th} term

let $x=3$

$\sum_{n=0}^{\infty}\left(\frac{3}{3}\right)^n$

$\sum_{n=0}^{\infty}(1)^n$

diverges by n^{th} term

$\boxed{f(x):(-3,3)}$

$$\boxed{\begin{array}{l} f'(x)=\sum_{n=0}^{\infty}\frac{n}{3}\left(\frac{x}{3}\right)^{n-1} : (-3,3) \\ f''(x)=\sum_{n=0}^{\infty}\frac{n(n-1)}{9}\left(\frac{x}{3}\right)^{n-2} : (-3,3) \\ \int f(x)dx=\sum_{n=0}^{\infty}\frac{\left(\frac{x}{3}\right)^{n+1}}{n+1} : [-3,3) \end{array}}$$

let $x=-3$

$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$

converges by alt test

let $x=3$

$\sum_{n=0}^{\infty}\frac{1}{n+1}$

diverges by p-series via limit comp.

46) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n5^n}$

$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{(n+1)5^{n+1}}\cdot\frac{n5^n}{(x-5)^n}\right|$

$\lim_{n\to\infty}\left|\frac{(x-5)n}{(n+1)5}\right|=\left|\frac{x-5}{5}\right|$

$-1<\frac{x-5}{5}<1$

$-5<x-5<5$

$0<x<10$ $\boxed{f(x):(0,10]}$

let $x=0$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-5)^n}{n5^n}$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n5^n}{n5^n}$

$\sum_{n=1}^{\infty}\frac{-1}{n}$ diverges by p-series

let $x=10$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(5^n)}{n5^n}$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$

converges by alt series

$f'(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n(x-5)^{n-1}}{n5^n}$

$f'(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^{n-1}}{5^n}$

let $x=10$ $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(5)^{n-1}}{5^n}$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{5}$ diverges by n^{th} term

$\boxed{f'(x):(0,10)}$

$f''(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n-1)(x-5)^{n-2}}{5^n}$ $\boxed{f''(x):(0,10)}$

$\int f(x)dx=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^{n+1}}{n(n+1)5^n}$ $\boxed{\int f(x)dx:[0,10]}$

let $x=0$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-5)^{n+1}}{n(n+1)5^n}$

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^{n+1}5^{n+1}}{(n^2+n)5^n}$

$\sum_{n=1}^{\infty}\frac{5}{n^2+n}$ converges by p-series via comparison

47) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

$\lim_{n\to\infty} \left| \frac{(x-1)^{n+2}}{n+2} \cdot \frac{n+1}{(x-1)^{n+1}} \right|$

$\lim_{n\to\infty} \left| \frac{(x-1)(n+1)}{n+2} \right| = |x-1|$

$|x-1| < 1$

$-1 < x-1 < 1$

$0 < x < 2$

let $x=0$

$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{n+1}$

$\sum_{n=0}^{\infty} \frac{1}{n+1}$

diverges by p-series

let $x=2$

$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(1)^{n+1}}{n+1}$

$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$

converges by alt. series

$\boxed{f(x): (0,2]}$

$f'(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(n+1)(x-1)^n}{n+1}$

$f'(x) = \sum_{n=0}^{\infty} (-1)^{n+1}(x-1)^n$

let $x=2$ $\sum_{n=0}^{\infty} (-1)^{n+1}(1)^n$

diverges by n^{th} term

$\boxed{f'(x): (0,2)}$

$f''(x) = \sum_{n=0}^{\infty} (-1)^{n+1} n(x-1)^{n-1}$ $\boxed{f''(x): (0,2)}$

$\int f(x)dx = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+2}}{(n+1)(n+2)}$

let $x=0$ $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(-1)^{n+2}}{n^2+3n+2}$

$\sum_{n=0}^{\infty} \frac{-1}{n^2+3n+2}$

converges by p-series

$\boxed{\int f(x)dx: [0,2]}$

73) False. It would converge at $x=-2$ if $R>2$. If $R=2$ we don't have a way to check the endpoint to see if it is true.

74) False. One end of the interval cannot be infinity without the other being infinity too ($R=\infty$).

75) True. This is just a horizontal shift right one.