Calculus Section 9.8 Radius and Interval of Convergence II

-Determine the endpoint convergence of a power series
-Differentiate and integrate a power series

Homework: page 654 #'s 45 – 47, 73 – 75

Derivative and Integral of a Power Series
In order to find the derivative or integral of a power series, derive or integrate with respect to x using the respective power rules.

For each derivative or antiderivative or a power series, the radius and interval of convergence will remain the same. The only change may be whether the endpoints are included/excluded.

Integrating: brackets remain **Differentiating: parenthesis remain**

Examples) Find the Interval of Convergence for f(x), f'(x), f''(x), and $\int f(x)dx$

1) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1} = f(x)$

$$\lim_{n\to\infty}\left|\frac{x^{n+2}}{n+2}\cdot\frac{n+1}{x^{n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{x(n+1)}{n+2}\right| = |x|$$

$$|x| < 1$$

$$-1 < x < 1$$

let $x = -1$

$$\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty}\frac{-1}{n+1}$$

diverges by p-series

let $x=1$

$$\sum_{n=0}^{\infty}\frac{(-1)^n(1)^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$$

converges by alt. series

$$\boxed{f(x) = (-1, 1]}$$

$$f'(x) = \sum_{n=0}^{\infty}\frac{(-1)^n(n+1)x^n}{(n+1)}$$

$$f'(x) = \sum_{n=0}^{\infty}(-1)^n x^n$$

let $x=1$ $\displaystyle\sum_{n=0}^{\infty}(-1)^n(1)^n$

diverges by n^{th} term

$$\boxed{f'(x): (-1, 1)}$$

$$f''(x) = \sum_{n=0}^{\infty}(-1)^n n x^{n-1}$$

$$\boxed{f''(x) = (-1, 1)}$$

$$\int f(x)dx = \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+2}}{(n+1)(n+2)}$$

let $x=-1$ $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^{n+2}}{n^2+3n+2}$

$$\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2}$$

converges by p-series

$$\boxed{\int f(x)dx: [-1, 1]}$$

2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}(x-2)^n}{n}$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{n+1} \cdot \frac{n}{(x-2)^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-2)n}{n+1} \right| = |x-2|$$

$$|x-2| < 1$$

$$-1 < x-2 < 1$$

$$1 < x < 3$$

let $x=1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+2}(-1)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges by p-series

let $x=3$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+2}(1)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n}$$

converges by alt. series

$$\boxed{f(x): (1,3]}$$

$$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n (x-2)^{n-1}}{n}$$

$$f'(x) = \sum_{n=1}^{\infty} (-1)^{n+1}(x-2)^{n-1}$$

$$\boxed{f'(x): (1,3)}$$

let $x=3$ $\sum_{n=1}^{\infty} (-1)^{n+1}(1)^{n-1}$

diverges by n^{th} term

$$f''(x) = \sum_{n=1}^{\infty} (-1)^{n+1}(n-1)(x-2)^{n-2}$$

$$\boxed{f''(x): (1,3)}$$

$$\int f(x)dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+2}(x-2)^{n+1}}{n(n+1)}$$

$$\boxed{\int f(x)dx: [1,3]}$$

let $x=1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+2}(-1)^{n+1}}{n^2+n}$$

$$\sum_{n=1}^{\infty} \frac{-1}{n^2+n}$$

converges by p-series