9.8 Radius and Interval of Convergence

Pg. 654 #'s 11-25 odd

11) $\sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n$

$\lim_{n\to\infty} \left|\frac{x^{n+1}}{4^{n+1}} \cdot \frac{4^n}{x^n}\right|$

$\lim_{n\to\infty} \left|\frac{x}{4}\right| = \left|\frac{x}{4}\right| < 1$

$-1 < \frac{x}{4} < 1$

$-4 < x < 4$

$R=4$ center $=0$

let $x=-4$

$\sum_{n=0}^{\infty} \left(\frac{-4}{4}\right)^n = (-1)^n$

diverges by n^{th} term

let $x=4$

$\sum_{n=0}^{\infty} \left(\frac{4}{4}\right)^n = (1)^n$

diverges by n^{th} term

$\boxed{(-4,4)}$

13) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

$\lim_{n\to\infty} \left|\frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n}\right|$

$\lim_{n\to\infty} \left|\frac{nx}{n+1}\right| = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \left|\frac{x}{1}\right| = |x| < 1$

$-1 < x < 1$

$R=1$ center $=0$

let $x=-1$

$\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n} = \frac{(-1)^{2n}}{n}$

diverges by n^{th} term test

let $x=1$

$\sum_{n=1}^{\infty} \frac{(-1)^n(1)^n}{n} = \frac{(-1)^n}{n}$

converges by alt. test

$\boxed{(-1,1]}$

15) $\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$

$\lim_{n\to\infty} \left|\frac{x^{5(n+1)}}{(n+1)!} \cdot \frac{n!}{x^{5n}}\right|$

$\lim_{n\to\infty} \left|\frac{x^5}{n+1}\right| = \left|\frac{x^5}{\infty}\right| = 0 < 1$

Always true

$R=\infty$

$\boxed{(-\infty,\infty)}$

17) $\sum_{n=0}^{\infty} (2n)! \left(\frac{x}{3}\right)^n$

$\lim_{n\to\infty} \left|\frac{(2(n+1))!\, x^{n+1}}{3^{n+1}} \cdot \frac{3^n}{x^n (2n)!}\right|$

$\lim_{n\to\infty} \left|\frac{(2n+2)(2n+1)\, x}{3}\right| = \left|\frac{\infty x}{3}\right| = \infty$

$\infty < 1$

Never true

$R=0$

$\boxed{\text{The series only converges at } x=0}$

19) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{6^n}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{6^{n+1}} \cdot \frac{6^n}{x^n} \right|$

$\lim_{n\to\infty} \left| \frac{x}{6} \right| = \left| \frac{x}{6} \right| < 1$

$-6 < x < 6$

let $x = -6$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-6)^n}{6^n} = \frac{-(6)^n}{6^n} = -1$

diverges by n^{th} term

let $x = 6$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(6)^n}{6^n} = (-1)^{n+1}$

diverges by n^{th} term

$\boxed{(-6, 6)}$

21) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n 9^n}$

$\lim_{n\to\infty} \left| \frac{(x-4)^{n+1}}{(n+1)9^{n+1}} \cdot \frac{n 9^n}{(x-4)^n} \right|$

$\lim_{n\to\infty} \left| \frac{(x-4)n}{(n+1)\cdot 9} \right| = \left| \frac{x-4}{9} \right| < 1$

$-9 < x-4 < 9$

$-5 < x < 13$

$R = 9$ center $= 4$

let $x = -5$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-5-4)^n}{n 9^n} = \frac{(-1)^{n+1}(-9)^n}{n 9^n}$

$\sum_{n=1}^{\infty} \frac{-1}{n}$ diverges by n^{th} term

let $x = 13$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(13-4)^n}{n 9^n} = \frac{(-1)^{n+1} 9^n}{n 9^n}$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges by alt. test

$\boxed{(-5, 13]}$

23) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

$\lim_{n\to\infty} \left| \frac{(x-1)^{n+2}}{n+2} \cdot \frac{n+1}{(x-1)^{n+1}} \right|$

$\lim_{n\to\infty} \left| \frac{(x-1)(n+1)}{(n+2)} \right| = |x-1| < 1$

$-1 < x-1 < 1$

$0 < x < 2$

$R = 1$ center $= 1$

let $x = 0$

$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{n+1} = \frac{1}{n+1}$

diverges by p-series via comparison

let $x = 2$

$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(1)^{n+1}}{n+1} = \frac{(-1)^{n+1}}{n+1}$

converges by alt. series test

$\boxed{(0, 2]}$

25) $\sum_{n=1}^{\infty} \frac{(x-3)^{n-1}}{3^{n-1}}$

$\lim_{n\to\infty} \left| \frac{(x-3)^n}{3^n} \cdot \frac{3^{n-1}}{(x-3)^{n-1}} \right|$

$\lim_{n\to\infty} \left| \frac{x-3}{3} \right| = \left| \frac{x-3}{3} \right| < 1$

$-1 < \frac{x-3}{3} < 1$ $R = 3$ center $= 3$

$-3 < x-3 < 3 \to 0 < x < 6$

let $x = 0$

$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{3^{n-1}} = (-1)^{n-1}$

diverges by n^{th} term

let $x = 6$

$\sum_{n=1}^{\infty} \frac{(3)^{n-1}}{3^{n-1}} = 1$

diverges by n^{th} term

$\boxed{(0, 6)}$