# Calculus Section 9.8 Radius and Interval of Convergence

-Understand the definition of a power series

-Find the radius and interval of convergence of a power series

| Homework: page 654 #'s 11 – 25 odd |
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Approximating the value of a function is one of the main uses of power series. However, not all power series are created equally. Some may give you good approximations for any value of x while others may only give good approximations at or near the center. The **radius of convergence** for a power series tell you how far away from the center you can be and still find a good approximation.

**Convergence of a Power Series**

For a power series centered at c, precisely one of the following is true:

1) The radius of convergence is R = 0. The series converges only at c.

2) The radius of convergence is R > 0 such that the series converges absolutely for |x – c| < R, and diverges for |x – c| > R.

3) The radius of convergence is infinity. The series converges absolutely for all x.

In the 2nd scenario where R = finite number, the interval may have open or closed endpoints. Use any of the convergence tests from earlier in the chapter to determine whether an endpoint should be included.

*We find the radius of convergence by using the Ratio Test.

**Example) Find the Radius and Interval of Convergence**

1) $\sum_{n=0}^{\infty} n!\,x^n$

$$\lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{1}\cdot\frac{1}{n!\,x^n}\right|$$

$$\lim_{n\to\infty}|(n+1)x| = |\infty(x)| = \infty$$

$\infty < 1 \leftarrow$ Never true

$R=0$ center $=0$

$\boxed{\text{The series only converges at the center } x=0.}$

2) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$$\lim_{n\to\infty}\left|\frac{x^{2(n+1)+1}}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{x^{2n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{x^{2n+3}}{(2n+3)!}\cdot\frac{(2n+1)!}{x^{2n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{x^2}{(2n+3)(2n+2)}\right| = \left|\frac{x^2}{\infty}\right| = 0$$

$0 < 1 \leftarrow$ Always true

$R=\infty$ center $=0$

$\boxed{(-\infty,\infty)}$

3) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n2^n}$ center = 2

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n2^n}{(x-2)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x-2)n}{(n+1)2}\right| = \frac{\infty}{\infty}$$

l'Hop

$$\lim_{n\to\infty}\left|\frac{x-2}{2}\right| = \left|\frac{x-2}{2}\right|$$

$$\left|\frac{x-2}{2}\right| < 1$$

$$-1 < \frac{x-2}{2} < 1$$

$$-2 < x-2 < 2$$

$$0 < x < 4$$

$R = 2$ center = 2

let $x = 0$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(0-2)^n}{n2^n}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(-2)^n}{n2^n}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(-1)^n(2)^n}{n2^n}$$

$$\sum_{n=0}^{\infty}\frac{-1}{n}$$

diverges by p-series test

let $x = 4$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(4-2)^n}{n2^n}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}2^n}{n2^n}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n}$$

converges by alternating series test

$\boxed{(0,4]}$

4) $\sum_{n=0}^{\infty} \frac{(x-3)^{n+1}}{(n+1)4^{n+1}}$ center = 3

$$\lim_{n\to\infty}\left|\frac{(x-3)^{n+2}}{(n+2)4^{n+2}}\cdot\frac{(n+1)4^{n+1}}{(x-3)^{n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x-3)(n+1)}{(n+2)4}\right| = \frac{\infty}{\infty}$$

l'Hop

$$\lim_{n\to\infty}\left|\frac{x-3}{4}\right| = \left|\frac{x-3}{4}\right|$$

$$\left|\frac{x-3}{4}\right| < 1$$

$$-1 < \frac{x-3}{4} < 1$$

$$-4 < x-3 < 4$$

$$-1 < x < 7$$

$R = 4$ center = 3

let $x = -1$

$$\sum_{n=0}^{\infty}\frac{(-1-3)^{n+1}}{(n+1)4^{n+1}}$$

$$\sum_{n=0}^{\infty}\frac{(-4)^{n+1}}{(n+1)4^{n+1}}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$$

converges by alt. series test

let $x = 7$

$$\sum_{n=0}^{\infty}\frac{(7-3)^{n+1}}{(n+1)4^{n+1}}$$

$$\sum_{n=0}^{\infty}\frac{4^{n+1}}{(n+1)4^{n+1}}$$

$$\sum_{n=0}^{\infty}\frac{1}{n+1}$$

diverges by p-series via limit comp.

$\boxed{[-1,7)}$