9.9 Geometric Power Series

Pg 662 #'s 5, 7, 9, 11, 12, 35, 36

5) $f(x) = \frac{1}{3-x}$, $c = 1$

$f(x) = \frac{1}{3-1-(x-1)}$

$f(x) = \frac{1}{2-(x-1)}$

$f(x) = \frac{1/2}{1-\frac{1}{2}(x-1)}$

$\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x-1}{2}\right)^n = \boxed{\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^{n+1}}}$

$r = \left|\frac{x-1}{2}\right| < 1$

$-1 < \frac{x-1}{2} < 1$

$-2 < x-1 < 2$

$-1 < x < 3$

$\boxed{\text{Interval}: (-1, 3)}$

7) $f(x) = \frac{1}{1-3x}$, $c = 0$

$\boxed{\sum_{n=0}^{\infty} 1(3x)^n}$

$|3x| < 1$

$-1 < 3x < 1$

$-\frac{1}{3} < x < \frac{1}{3}$

$\boxed{\text{Interval}: (-\frac{1}{3}, \frac{1}{3})}$

9) $g(x) = \frac{5}{2x-3}$, $c = -3$

$g(x) = \frac{5}{-3+2x}$

$g(x) = \frac{5}{-3-6+2(x+3)}$

$g(x) = \frac{5}{-9+2(x+3)}$

$g(x) = \frac{-5/9}{1-\frac{2}{9}(x+3)}$

$\boxed{\sum_{n=0}^{\infty} \left(-\frac{5}{9}\right)\left(\frac{2(x+3)}{9}\right)^n}$

$r = \left|\frac{2(x+3)}{9}\right| < 1$

$-1 < \frac{2(x+3)}{9} < 1$

$-4.5 < x+3 < 4.5$

$-7.5 < x < 1.5$

$\boxed{\text{Interval}: (-7.5, 1.5)}$

11) $f(x) = \frac{3}{4+3x}$, $c = 0$

$f(x) = \frac{3/4}{1+\frac{3}{4}x}$

$f(x) = \frac{3/4}{1--\frac{3}{4}x}$

$\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)\left(\frac{-3x}{4}\right)^n$

$\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{3^{n+1}x^n}{4^{n+1}}}$

$r = \left|\frac{-3x}{4}\right| < 1$

$-1 < \frac{3x}{4} < 1$

$-\frac{4}{3} < x < \frac{4}{3}$

$\boxed{\text{Interval}: (-\frac{4}{3}, \frac{4}{3})}$

12) $f(x) = \frac{4}{2+3x}$, $c=3$

$f(x) = \frac{4}{2+9+3(x-3)}$

$f(x) = \frac{4}{11+3(x-3)}$

$f(x) = \frac{4/11}{1 - -\frac{3}{11}(x-3)}$

$\sum_{n=0}^{\infty} \left(\frac{4}{11}\right)\left(\frac{-3(x-3)}{11}\right)^n$

$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 4(3)^n}{11^{n+1}} (x-3)^n}$

$r = \left|\frac{-3(x-3)}{11}\right| < 1$

$-1 < \frac{3(x-3)}{11} < 1$

$-\frac{11}{3} < x-3 < \frac{11}{3}$

$-\frac{2}{3} < x < \frac{20}{3}$

$\boxed{\text{Interval: } \left(-\frac{2}{3}, \frac{20}{3}\right)}$

35) $f(x) = \frac{1}{(1-x)^2}$

$\frac{d}{dx}\left[\frac{1}{1-x}\right] = \frac{1}{(1-x)^2}$

$\frac{d}{dx}\left[\sum_{n=0}^{\infty} x^n\right] = \sum_{n=0}^{\infty} n x^{n-1}$

$\boxed{\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} n x^{n-1}}$

36) $f(x) = \frac{x}{(1-x)^2}$

$\frac{x}{(1-x)^2} = x\left(\frac{1}{(1-x)^2}\right)$

$\frac{x}{(1-x)^2} = \sum_{n=0}^{\infty} x(n x^{n-1})$

$\boxed{\frac{x}{(1-x)^2} = \sum_{n=0}^{\infty} n x^n}$