

### 3.1 – 3.6 Practice AP Questions

Name: Answer Key

1) The graph of  $y = 3x^2 - x^3$  has a relative maximum at

(A) (0,0) only

(B) (1,2) only

**(C) (2,4) only**

(D) (4,-16) only

(E) (0,0) and (2,4)

$$y' = 6x - 3x^2$$

$$0 = 3x(2-x)$$

$$x=0 \quad x=2$$

x	-1	0	1	2	3
f'(x)	-	0	+	0	-

2) If the graph of  $f(x) = 2x^2 + k/x$  has a point of inflection at  $x = -1$ , then the value of  $k$  is

(A) -2

(B) -1

(C) 0

(D) 1

**(E) 2**

$$f(x) = 2x^2 + kx^{-1}$$

$$f'(x) = 4x - kx^{-2}$$

$$f''(x) = 4 + 2kx^{-3}$$

$$0 = 4 + \frac{2k}{x^3}$$

$$-4 = \frac{2k}{x^3}$$

$$-4x^3 = 2k$$

$$-2x^3 = k$$

$$-2(-1)^3 = k$$

$$2 = k$$

3) What are all value of  $x$  for which the graph of  $y = \frac{2}{4-x}$  is concave downward?

(A) No values of  $x$

(B)  $x < 4$

(C)  $x > -4$

(D)  $x < -4$

**(E)  $x > 4$**

$$y = 2(4-x)^{-1}$$

$$y' = -2(4-x)^{-2}(-1)$$

$$y' = 2(4-x)^{-2}$$

$$y'' = -4(4-x)^{-3}(-1)$$

$$y'' = \frac{4}{(4-x)^3}$$

if  $(4-x)^3$  is negative,  
then  $\frac{4}{(4-x)^3}$  is negative

$$(4-x)^3 < 0$$

$$4-x < 0$$

$$4 < x$$

4) The functions  $f$  and  $g$  are piecewise linear functions whose graph are shown below. If  $h(x) = f(x)g(x)$ , then

$h'(3) =$

(A)  $-8/3$

(B)  $-1/3$

**(C) 0**

(D)  $2/3$

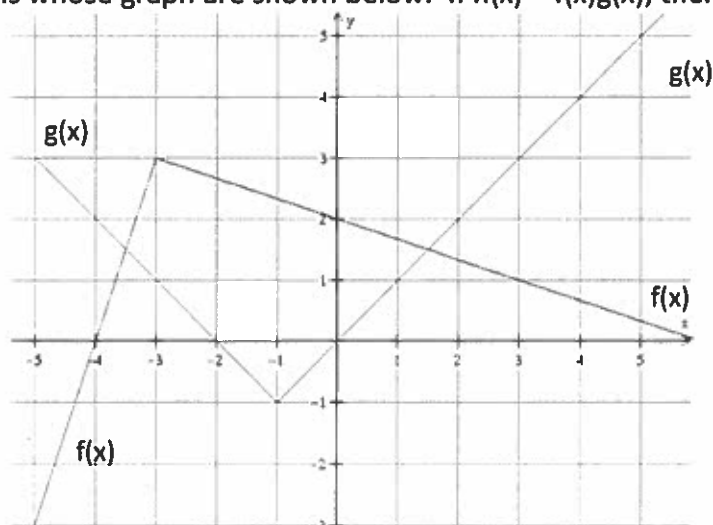
(E)  $8/3$

$$h'(x) = f(x)g'(x) + g(x)f'(x)$$

$$h'(3) = (1)(1) + (3)(-1/3)$$

$$h'(3) = 1 - 1$$

$$h'(3) = 0$$



5) At what value(s) of  $x$  does  $f(x) = x^4 - 8x^2$  have a relative minimum?

(A) 0 and -2 only

(B) 0 and 2 only

(C) 0 only

(D) -2 and 2 only

(E) -2, 0, and 2

$$f'(x) = 4x^3 - 16x$$

$$0 = 4x(x^2 - 4)$$

$$0 = 4x(x+2)(x-2)$$

$$x = 0 \quad x = -2 \quad x = 2$$

$x$	-3	-2	-1	0	1	2	3
$f'(x)$	-	0	+	0	-	0	+

6) The function  $y = x^4 + bx^2 + 8x + 1$  has a horizontal tangent and a point of inflection for the same value of  $x$ . What must be the value of  $b$ ?

(A) -6

(B) -1

(C) 1

(D) 4

(E) 6

$$y' = 4x^3 + 2bx + 8$$

$$y'' = 12x^2 + 2b$$

$$0 = 12x^2 + 2b$$

$$-12x^2 = 2b$$

$$4x^3 + 2bx + 8 = 12x^2 + 2b$$

$$4x^3 + (-12x^2)x + 8 = 12x^2 + (-12x^2)$$

$$4x^3 - 12x^3 + 8 = 0$$

$$-8x^3 = -8$$

$$x^3 = 1$$

$$x = 1$$

$$-12x^2 = 2b$$

$$-6x^2 = b$$

$$-6(1)^2 = b$$

$$-6 = b$$

7) Let  $f$  be the function given by  $f(x) = x^3$ . What are all value of  $c$  that satisfy the conclusion of the Mean Value Theorem on the closed interval  $[-1, 2]$ ?

(A) 0 only

(B) 1 only

(C)  $\sqrt{3}$

(D) -1 and 1

(E)  $-\sqrt{3}$  and  $\sqrt{3}$

$$f(2) = 8 \quad f(-1) = -1$$

$$f'(c) = \frac{8 - (-1)}{2 - (-1)} = \frac{9}{3} = 3$$

$$f'(x) = 3x^2$$

$$3 = 3x^2$$

$$1 = x^2$$

$$x = \pm 1$$

$c$  must be in the open interval  $(-1, 2)$

8) What are all values of  $x$  for which the function  $f(x) = x^3 + 6x^2 + 9x + 1$  is increasing?

(A)  $(-\infty, -3)$  only

(B)  $(-3, -1)$  only

(C)  $(-1, \infty)$  only

(D)  $(-\infty, -3) \cup (-1, \infty)$

(E)  $(-\infty, -3) \cup (1, \infty)$

$$f'(x) = 3x^2 + 12x + 9$$

$$0 = 3(x^2 + 4x + 3)$$

$$0 = (x+3)(x+1)$$

$$x = -3 \quad x = -1$$

$x$	-4	-3	-2	-1	0
$f'(x)$	+	0	-	0	+

9) If  $f$  is defined by  $f(x) = \frac{5x^7}{7} + 4x^6 + 6x^5 + x + 1$ , what are all the  $x$ -coordinates of the points of inflection of the graph of  $f$ ?

(A) -2 only

(B) 0 only

(C) 2 only

(D) -2 and 0 only

(E) -2, 0, 2

$$f'(x) = 5x^6 + 24x^5 + 30x^4 + 1$$

$$f''(x) = 30x^5 + 120x^4 + 120x^3$$

$$0 = 30x^3(x^2 + 4x + 4)$$

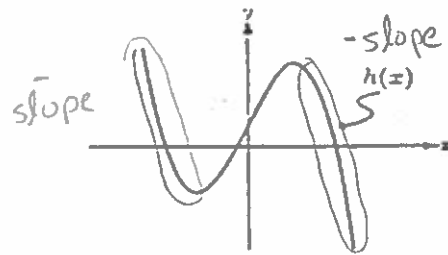
$$0 = 30x^3(x+2)(x+2)$$

$$x=0 \quad x=-2$$

$x$	-3	-2	-1	0	1
$f''(x)$	-	0	-	0	+

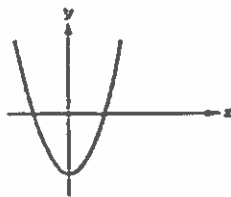
No change in concavity

10)

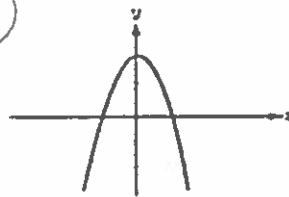


The graph of  $h(x)$  is shown above. Which of the following could be the graph of  $y = h'(x)$ ?

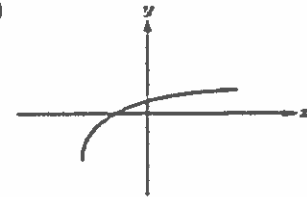
(A)



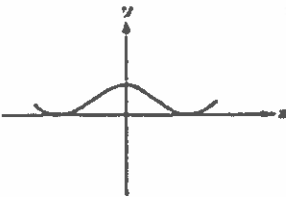
(B)



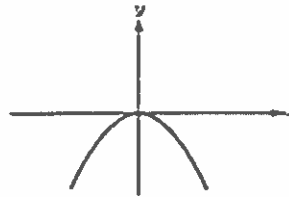
(C)



(D)



(E)

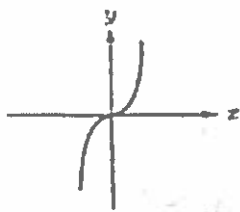


11. If, for all real numbers  $x$ ,  $f'(x) < 0$  and  $f''(x) > 0$ , which of the following curves could be part of the graph of  $f$ ?

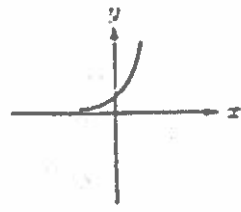
decreasing

concave up

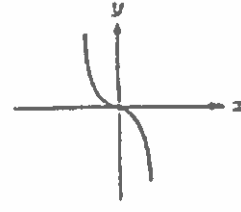
(A)



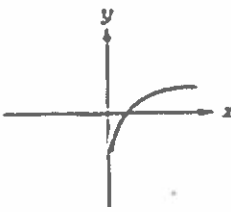
(B)



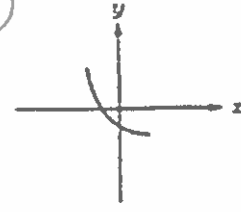
(C)



(D)

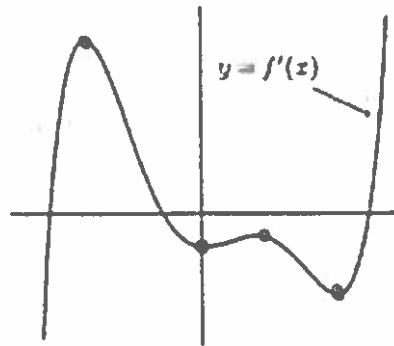


(E)



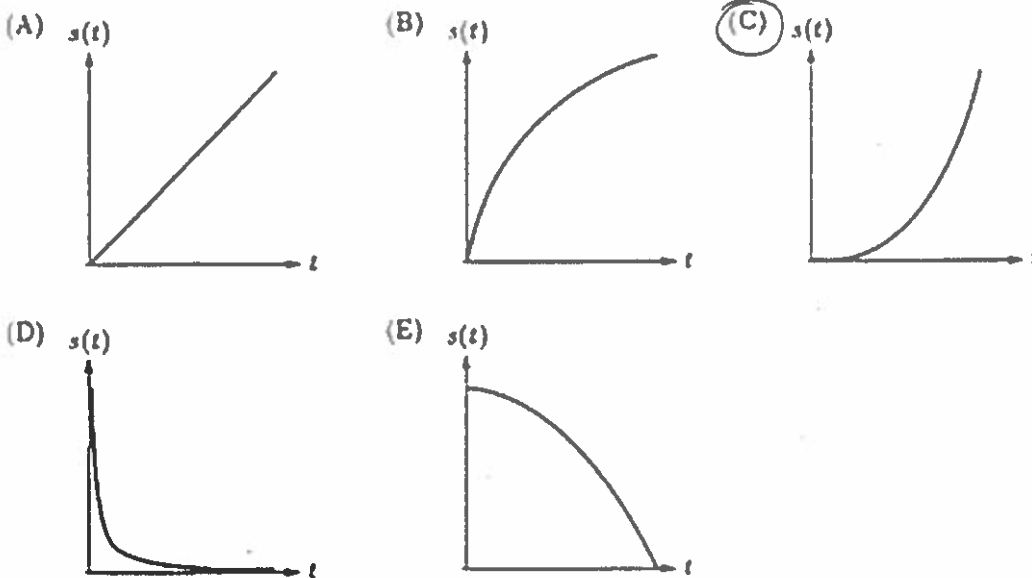
12) The figure below shows the graph of the derivative of a function  $f$ . How many points of inflection does  $f$  have in the interval shown?

- (A) None
- (B) One
- (C) Two
- (D) Three

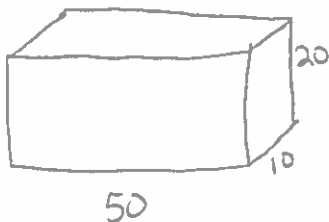


(E) Four

13) Which graph best represents the position of a particle,  $s(t)$ , as a function of time, if the particle's velocity and acceleration are both positive?



14) Water is draining out of a rectangular tank whose base measures  $50 \times 10$  cm and height measures 20 cm. The water level of the tank is changing by  $0.1$  cm every second. The water is draining into another rectangular tank whose base measures  $30 \times 20$  cm and height measures 20 cm. How fast is the water level rising in the 2<sup>nd</sup> tank?



1<sup>st</sup> Tank

$$V = Bh$$

$$\frac{dV}{dt} = B \frac{dh}{dt} + h \frac{dB}{dt}$$

$$\frac{dV}{dt} = 500 \frac{dh}{dt} + 20(0)$$

$$\frac{dV}{dt} = 500(-.1)$$

$$\frac{dV}{dt} = -50 \text{ cm}^3/\text{s}$$

2<sup>nd</sup> Tank

$$V = Bh$$

$$\frac{dV}{dt} = B \frac{dh}{dt} + h \frac{dB}{dt}$$

$$50 = 600 \frac{dh}{dt} + 20(0)$$

$$\frac{50}{600} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{12} \text{ cm/s}$$

