AP Questions 9.1 – 9.6

Name: Answer Key

1) What is the sum of the series $\sum_{n=0}^{\infty} \frac{2+3^n}{5^n}$ $= 2\left(\frac{1}{5}\right)^n + \left(\frac{3}{5}\right)^n$

$\frac{2}{1-1/5} + \frac{1}{1-3/5} = \frac{2}{4/5} + \frac{1}{2/5}$

$\frac{10}{4} + \frac{5}{2} = \frac{10}{2} = 5$

(A) 15/4 (B) 25/6 (C) 9/2 (D) 5 (E) Divergent

2) $\sum_{n=2}^{\infty} \frac{3}{5^n} = 3\left(\frac{1}{5}\right)^n$ $\frac{3}{1-1/5} = \frac{3}{4/5} = \frac{15}{4}$ $n=0$ 3 $n=1$ $\frac{3}{5}$

$\frac{15}{4} - 3 - \frac{3}{5}$

$\frac{75}{20} - \frac{60}{20} - \frac{12}{20} = \frac{3}{20}$

(A) 3/20 (B) 9/20 (C) 9/10 (D)5/2 (E) 15/4

3) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n} = \left(\left(\frac{1}{2}\right)^2\right)^n = \left(\frac{1}{4}\right)^n$ $\frac{1}{1-1/4} = \frac{1}{3/4} = \frac{4}{3}$ $n=0$ 1 $\frac{4}{3} - 1 = \frac{1}{3}$

(A) 1/3 (B) ½ (C) 1 (D) 2 (E) Infinity

4) Which of the following are true statements?

✓ I. If $\sum_{n=0}^{\infty} a_n$ converges conditionally, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

✓ II. If $\sum_{n=0}^{\infty} |a_n|$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.

X III. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges Missing 2nd condition of alternating series test

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

5) The sum of the infinite geometric series $2-\frac{2}{3}+\frac{2}{9}-\frac{2}{27}+\ldots$ is $\sum_{n=0}^{\infty} 2\left(\frac{-1}{3}\right)^n$ $\frac{2}{1-(-1/3)} = \frac{2}{4/3} = \frac{6}{4} = \frac{3}{2}$

(A)-6 (B) -3 (C) 0 (D)3/7 (E) 3/2

6) Which of the following series are convergent?

X I. $\sum_{n=1}^{\infty}(-1)^{n+1}$ $1-1+1-1+1-1\cdots$

X II. $\sum_{n=1}^{\infty}(-1)^{n+1}n$ $1-2+3-4+5-6+\ldots$

X III. $\sum_{n=1}^{\infty}\left(\frac{1+n}{n}\right)^n$ $\frac{1}{n} < \left(\frac{1+n}{n}\right)^n$ diverges by direct comparison

(A) None (B) II only (C) III only (D) I and II (E) I and III

7) Which of the following functions grow faster than e^x as $x \to \infty$?

$e = 2.718$

(A) x^4 (B) $\ln x$ (C) e^{-x} (D) 3^x (E) $\frac{1}{2}e^x$

8) Let a_n, b_n, and c_n be sequences of positive numbers such that for all positive intergers n, $a_n \le b_n \le c_n$. If $\sum_{n=1}^{\infty} b_n$ converges, then which of the following statements must be true?

✓ I. $\sum_{n=1}^{\infty} a_n$ converges

X II. $\sum_{n=1}^{\infty} c_n$ converges maybe

✓ III. $\sum_{n=1}^{\infty}(a_n + b_n)$ converges

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III