AP Questions Parametric, Vector, Polar

Name: Answer Key

1) A particle moves in the xy-plane so that its velocity vector at time *t* is v(t) = (t^2, sinπt) and the particle's position vector at time t = 0 is (1,0). What is the position vector of the particle when t = 3?

$x(t)=\frac{1}{3}t^3+C \quad 1=\frac{1}{3}(0)^3+C \quad C=1 \quad x(t)=\frac{1}{3}t^3+1 \quad x(3)=9+1=10$

(A) (9, 1/π) (B) (10, 2/π) (C) (6, -2π) (D) (10, 2π) (E) (10, 2)

$y(t)=\frac{-1}{\pi}\cos(\pi t)+C \quad 0=\frac{-1}{\pi}\cos(0)+C \quad C=\frac{1}{\pi} \quad y(t)=\frac{-1}{\pi}\cos(\pi t)+\frac{1}{\pi} \quad y(3)=\frac{-1}{\pi}\cos(3\pi)+\frac{1}{\pi}$

$y(3)=\frac{1}{\pi}+\frac{1}{\pi}=\frac{2}{\pi}$

2) Which of the following is an equation of the line tangent to the curve with parametric equations x = $3e^{-t}$, y = $6e^{t}$ at the point where t = 0?

$\frac{dy}{dt}=6e^t \quad \frac{dx}{dt}=-3e^{-t} \quad x(0)=3 \quad y(0)=6$

(A) 2x + y – 12 = 0

(B) -2x + y – 12 = 0

(C) 2x + y – 6 = 0

(D) -2x + y – 6 = 0

(E) 2x + y = 0

$\frac{dy}{dx}=\frac{6e^t}{-3e^{-t}}=-2e^{2t}$

$\frac{dy}{dx}=-2e^{2(0)}=-2$

$y-6=-2(x-3)$

$y-6=-2x+6$

$y+2x-12=0$

3) A particle moves on the x-axis so that at any time t its velocity v(t) = sin2t subject to the condition x(0) = 0 where x(t) is the position function. Which of the following is an expression for x(t)?

(A) cost2t + ½

(B) -½sin2t + ½

(C) -½cos2t

(D) -½cos2t + ½

(E) -½cos2t – ½

$x(t)=\frac{-1}{2}\cos(2t)+C \qquad x(t)=\frac{-1}{2}\cos(2t)+\frac{1}{2}$

$x(0)=\frac{-1}{2}\cos(0)+C$

$0=-\frac{1}{2}(1)+C$

$1/2=C$

4) (calc) Which of the following gives the area of the region enclosed by the graph of the polar curve r = 1 + cosθ?

(A) $\int_0^{\pi}(1+\cos^2\theta)d\theta$ (B) $\int_0^{\pi}(1+\cos\theta)^2 d\theta$ (C) $\int_0^{2\pi}(1+\cos\theta)d\theta$

(D) $\int_0^{2\pi}(1+\cos\theta)^2 d\theta$ (E) $\frac{1}{2}\int_0^{2\pi}(1+\cos^2\theta)d\theta$

5) The curve in the xy-plane is defined parametrically by the equation x = t^2 + t and y = t^2 – t. For what values of t is the tangent line to the curve horizontal?

$\frac{dx}{dt}=2t+1 \quad \frac{dy}{dt}=2t-1$

(A) t = -1 (B) t = -½ (C) t = 0 (D) t = ½ (E) t = 1

$\frac{2t-1}{2t+1}=0 \quad 2t-1=0 \quad 2t=1 \quad t=1/2$

2015 #2 (Calculator)

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos(t^2), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

a) $\int_1^2 \cos(t^2)\,dt = x(2) - x(1)$

$x(2) = \int_1^2 \cos(t^2)\,dt + x(1)$

$\boxed{x(2) = 2.557}$

b) $\frac{dy}{dx} = \frac{e^{.5t}}{\cos(t^2)}$

$2 = \frac{e^{.5t}}{\cos(t^2)}$

$\boxed{t = .840}$

c) speed $= \sqrt{(\cos(t^2))^2 + (e^{.5t})^2}$

$3 = \sqrt{(\cos(t^2))^2 + (e^{.5t})^2}$

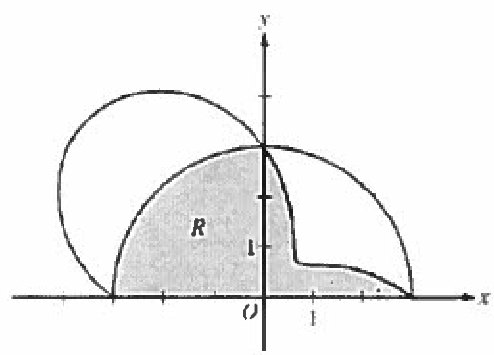
$\boxed{t = 2.196}$

d) $d = \int_0^1 \sqrt{(\cos(t^2))^2 + (e^{.5t})^2}\,dt$

$\boxed{d = 1.595}$

2014 #2 (Calculator)

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.



(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

a) $3 = 3 - \sin(2\theta)$

$0 = \sin(2\theta)$

$2\theta = 0, \pi, 2\pi$

$\theta = 0, \frac{\pi}{2}, \pi$

$R = \frac{1}{2}\int_0^{\pi/2} (3 - 2\sin(2\theta))^2\,d\theta + \frac{1}{2}\int_{\pi/2}^{\pi} (3)^2\,d\theta = \boxed{9.708}$

b) $x = r\cos\theta$

$\frac{dx}{d\theta} = r(-\sin\theta) + \frac{dr}{d\theta}\cos\theta$

$\frac{dx}{d\theta} = (3 - 2\sin(2\theta))(-\sin\theta) + (-4\cos(2\theta))\cos(\theta)$

$\boxed{\frac{dx}{d\theta} = -2.366}$

c) $d = 3 - (3 - 2\sin(2\theta))$

$d = 2\sin(2\theta)$

$\frac{dd}{d\theta} = 4\cos(2\theta)$

$\boxed{\frac{dd}{d\theta} = -2}$

d) $r = 3 - 2\sin(2\theta)$

$\frac{dr}{dt} = -4\cos(2\theta)\frac{d\theta}{dt}$

$\frac{dr}{dt} = -4\cos(2(\frac{\pi}{6}))(3)$

$\boxed{\frac{dr}{dt} = -6}$