Practice AP Questions 2.1-2.4

Name: Answer Key

1) If f(x) = $\sqrt{4\sin x+2}$, then f'(0) =

(A) -2 (B) 0 (C) 1 (D) $\sqrt{2}/2$ (E) $\sqrt{2}$

$f(x)=(4\sin x+2)^{1/2}$

$f'(x)=\frac{1}{2}(4\sin x+2)^{-1/2}\cdot 4\cos x$

$f'(0)=\frac{1}{2}(4(0)+2)^{-1/2}\cdot 4(1)$

$f'(0)=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)\cdot 4=\frac{2}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}$

2) A particle moves along the x-axis in such a way that its position at time t is given by $x(t)=\frac{1-t}{1+t}$. What is the acceleration of the particle at time t=0?

(A) -4

(B) -2

(C) -3/5

(D) 2

(E) 4

$v(t)=\frac{(1+t)(-1)-(1-t)(1)}{(1+t)^2}=\frac{-1-t-1+t}{(1+t)^2}=\frac{-2}{(1+t)^2}=-2(1+t)^{-2}$

$a(t)=4(1+t)^{-3}\cdot(1)=\frac{4}{(1+t)^3}$

$a(0)=\frac{4}{(1+0)^3}=\frac{4}{1}=4$

calc

3) Let f be the function given by f(x) = tanx and let g be the function given by g(x) = x^2. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines?

(A) 0

(B) 0.660

(C) 2.083

(D) 2.194

(E) 2.207

$f'(x)=\sec^2 x$ $g'(x)=2x$

$\sec^2 x=2x$

4) Two functions f(x) and g(x) are differentiable. If h(x) = x^2g(x) – f(3x + 1), determine the value of h'(x).

(A) 2xg'(x) – 3f'(3x + 1)

$x^2 g'(x)+g(x)2x-f'(3x+1)\cdot 3$

(B) 2xg'(x) – f'(3)

(C) x^2g'(x) + 2xg(x) – f'(3x + 1)

(D) x^2g'(x) + 2xg(x) – 3f'(3x + 1)

(E) x^2g'(x) – f'(3x + 1)

5) Let f and g be differentiable functions such that

$$f(1) = 4,\ g(1) = 3,\ f'(3) = -5$$
$$f'(1) = -4,\ g'(1) = -3,\ g'(3) = 2$$

If h(x) = f(g(x)), then h'(1) =

$h'(x) = f'(g(x))\, g'(x)$

$h'(1) = f'(g(1))\, g'(1)$

$h'(1) = f'(3) \cdot (-3)$

(A) -9 (B) 15 (C) 0 (D) -5 (E) -12

$h'(1) = (-5) \cdot (-3)$

$h'(1) = 15$

6) If f(x) = $(2 + 3x)^4$, then the fourth derivative of f is

(A) 0 (B) 4!(3) (C) $4!(3^4)$ (D) $4!(3^5)$ (E) 4!(2 + 3x)

$f'(x) = 4(2+3x)^3 \cdot 3$

$f''(x) = 4 \cdot 3\,(2+3x)^2 \cdot 3^2$

$f'''(x) = 4 \cdot 3 \cdot 2\,(2+3x)^1 \cdot 3^3$

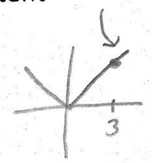
$f^{(4)}(x) = 4 \cdot 3 \cdot 2 \cdot 1\,(2+3x)^0 \cdot 3^4$

$= 4!\ 3^4$

7) The $\lim_{h \to 0} \dfrac{|x+h| - |x|}{h}$ at x = 3 is

(A) -1 (B) 0 (C) 1 (D) 3 (E) nonexistent

$f(x) = |x|$ $f'(3) =$ slope of $|x|$ at $x = 3$



8) If y = 7 is a horizontal asymptote of a rational function f, then which of the following must be true?

(A) $\lim_{x \to 7} f(x) = \infty$ (B) $\lim_{x \to -\infty} f(x) = -7$ (C) $\lim_{x \to 0} f(x) = 7$ (D) $\lim_{x \to 7} f(x) = 0$ (E) $\lim_{x \to \infty} f(x) = 7$

9) Let f(x) be a continuous and differentiable function. The table below gives the values of f(x) and f'(x), the derivative of f(x), at several values. If g(x) = $\dfrac{1}{f(x)}$, what is the value of g'(2)?

$g(x) = (f(x))^{-1}$

$g'(x) = -1\,(f(x))^{-2} \cdot f'(x)$

$g'(x) = \dfrac{-1}{(f(x))^2} \cdot f'(x)$

(A) -1/8

(B) 0

(C) 1/16

(D) 1/64

(E) 16

x	1	2	3	4
f(x)	-3	-8	-9	0
f'(x)	-5	-4	3	16

$g'(2) = \dfrac{-1}{(f(2))^2} \cdot f'(2)$

$g'(2) = \dfrac{-1}{(-8)^2} \cdot (-4) = \dfrac{4}{64} = \dfrac{1}{16}$

10) If $f(x) = \cos^2(x)$, then $f''(\pi) =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 2π

$f(x) = (\cos x)^2$

$f'(x) = 2\cos x \cdot (-\sin x)$

$f''(x) = 2\cos x(-\cos x) + (-\sin x)(-2\sin x)$

$f''(x) = -2\cos^2 x - 2\sin^2 x$

$f''(\pi) = -2(-1)^2 - 2(0)^2 = -2$

calc

11) Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < x < 6$. For how many values of t do the particles have the same acceleration?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$y_1 = \cos 2t$

$y_1' = -2\sin 2t$

$y_1'' = -4\cos 2t$

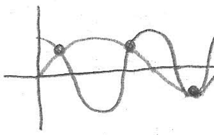
$y_2 = 4\sin t$

$y_2' = 4\cos t$

$y_2'' = -4\sin t$

$-4\cos 2t = -4\sin t$

$\cos 2t = \sin t$



12) Evaluate $\lim\limits_{h \to 0} \dfrac{5\left(\frac{1}{2}+h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

$f(x) = 5x^4$

$f'(x) = 20x^3$

(A) 5/2 (B) 5/16 (C) 40 (D) 160 (E) The limit DNE

$f'(\tfrac{1}{2}) = 20(\tfrac{1}{2})^3$

$= 20(\tfrac{1}{8})$

$= \tfrac{5}{2}$

13) If f is continuous on [2, 6], with f(2) = 20 and f(6) = 10, then the Intermediate Value Theorem says which of the following is true?

~~I.~~ f(x) = 25 does not have a solution on [2, 6]

√ II. f(x) = 17 has a solution on [2, 6]

~~III.~~ f(x) = 0 has a solution on [2, 6]

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III