AP Questions Chapter 5 Name Answer Key

1) If $f(x)=e^{\sin x}$, how many zeros does f'(x) have on the closed interval [0, 2π]?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$e^{\sin x}$ is always positive

$f'(x) = \cos x\, e^{\sin x}$

$\cos x = 0$

$x = \frac{\pi}{2}$ $x = \frac{3\pi}{2}$

2) $\int \frac{1}{\sqrt{4-x^2}}\,dx =$

$u = x$ $a = 2$

$du = dx$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$

(A) $\arcsin\frac{x}{2}+C$

(B) $2\sqrt{4-x^2}+C$

(C) $\arcsin x + C$

(D) $\sqrt{4-x^2}+C$

(E) $\frac{1}{2}\arcsin\frac{x}{2}+C$

3) If y = $x(\ln x)^2$, then $\frac{dy}{dx}$ =

$\frac{dy}{dx} = x\left(2(\ln x)^1\left(\frac{1}{x}\right)\right) + (\ln x)^2(1)$

$\frac{dy}{dx} = 2\ln x + (\ln x)^2$

$\frac{dy}{dx} = \ln x(2+\ln x)$

(A) $3(\ln x)^2$

(B) $(\ln x)(2x + \ln x)$

(C) $(\ln x)(2 + \ln x)$

(D) $(\ln x)(2 + x\ln x)$

(E) $(\ln x)(1 + \ln x)$

4) $4\int_1^{e^2} \frac{x-x^3}{x^2}\,dx =$

$4\int_1^{e^2} \frac{1}{x}\,dx - 4\int_1^{e^2} x\,dx$

(A) $3 - e^2$ (B) $3 - e^4$ (C) $5 - e^2$ (D) $5 - e^4$ (E) $10 - 2e^4$

$4\ln|x|\Big|_1^{e^2} - 2x^2\Big|_1^{e^2}$

$4\ln|e^2| - 4\ln|1| - (2(e^2)^2 - 2(1)^2) \rightarrow 4(2) - 0 - (2e^4 - 2) \rightarrow 8 - 2e^4 + 2$

5) The function f(x) = tan(3^x) has one zero in the interval [0, 1.4]. The derivative at this point is (calc.)

$f'(x) = \sec^2(3^x) \cdot (\ln 3)3^x$

$0 = \tan(3^x)$

$x = 1.041978$

$f'(1.041978) = \sec^2(3^{1.042}) \cdot (\ln 3)3^{1.042}$

$f'(1.042) = 3.451$

(A) 0.411

(B) 1.042

(C) 3.451

(D) 3.763

(E) undefined

6) A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

a) $a(t) = \dfrac{-e^t}{e^{2t}+1}$

$\boxed{a(2) = -.133}$

b) $v(2) = -.436$

The speed is increasing because both $a(2)$ and $v(2)$ are negative.

c) $v(t) = 0$

$1 - \arctan(e^t) = 0$

$t = .44302272$

The velocity changes from positive to negative at this time meaning position is at a max.

d) $\int_0^2 (1-\arctan(e^t)) = x(2) - x(0)$

$-.361 = x(2) - (-1)$ $\boxed{x(2) = -1.361}$

The particle is moving away because velocity and position are both negative.

7)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

$h(1) = f(g(1)) - 6 = 3$

$h(3) = f(g(3)) - 6 = -7$

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

a) $h(r) = -5$ from $1 < r < 3$ because $h(3) \leq -5 \leq h(1)$ and h is continuous. This is because of the Intermediate Value Thm.

b) $\dfrac{h(3) - h(1)}{3-1}$

$\dfrac{-7-3}{3-1} = \dfrac{-10}{2} = -5$

By the Mean Value Thm, $h'(c) = -5$ on the interval.

c) $w'(x) = f(g(x))\,g'(x)$

$w'(3) = f(g(3))\,g'(3)$

$w'(3) = -1 \times 2$

$\boxed{w'(3) = -2}$

d)

$g(x)$	$g^{-1}(x)$
(?, 2)	(2, ?)
(1, 2)	(2, 1)

$g'(1) = 5 \rightarrow (g^{-1})'(2) = \frac{1}{5}$

$\boxed{y - 1 = \frac{1}{5}(x-2)}$