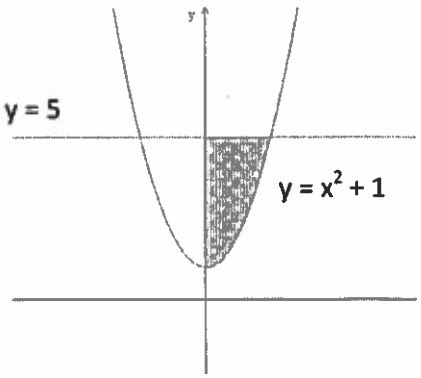
AP Questions Chapter 7

1) For the figure to the right, the area of the shaded region is



(A) 14/3 (B) 16/3 (C) 28/3

(D) 32/3 (E) 65/3

$$A = \int_0^2 (5 - (x^2 + 1))\,dx$$

$$A = 5.\overline{3}$$

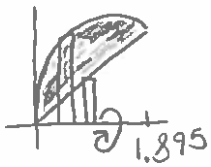
2) If, for all real numbers x, f(x) = g(x) + 5, then on any interval [a, b] the area of the region between the graphs of f(x) and g(x) is

(A) 5 (B) 5a + 5b (C) 5b – 5a (D) 5a – 5b (E) 5ab

$$\int_a^b \text{upper} - \text{lower}$$

$$\int_a^b ((g(x) + 5) - g(x)) \longrightarrow \int_a^b 5\,dx \rightarrow 5x\Big|_a^b \rightarrow 5b - 5a$$

3) The region in the first quadrant enclosed by the graphs y = x and y = 2sinx is revolved about the x-axis. The volume of the solid generated is



(A) 1.895

(B) 2.126

(C) 5.811

(D) 6.678

(E) 13.355

$$V = \pi \int_0^{1.895} (2\sin x)^2\,dx - \pi \int_0^{1.895} (x)^2\,dx$$

$$V = 6.678$$

4) The area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from x = 1 to x = 3 is

(A) 36 (B) 34 (C) 31 (D) 26 (E) 12

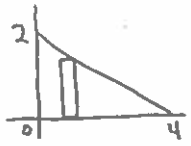
$$A = \int_1^3 (3x^2 + 2x)\,dx$$

$y = -\frac{1}{2}x + 2$

5) The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle?

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) $\frac{8\pi}{3}$

(D) $\frac{32\pi}{3}$ (E) $\frac{64\pi}{3}$



diameter $= -\frac{1}{2}x + 2$

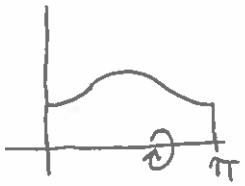
radius $= -\frac{1}{4}x + 1$

$V = \frac{1}{2}\pi \int_0^4 \left(-\frac{1}{4}x + 1\right)^2 dx$

$V = .6\pi$

6) The region in the first quadrant enclosed by the x-axis, the line $x = \pi$, and the curve $y = \cos(\cos(x))$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 1.921 (B) 3.782 (C) 6.040 (D) 8.130 (E) 23.781

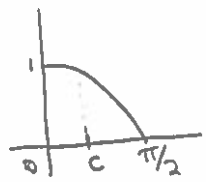


$V = \pi \int_0^{\pi} (\cos(\cos(x)))^2 dx$

$V = 1.922\pi$

7) The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = 0$ and $x = \pi/2$ is divided into two regions by the line $x = c$. If the area of the region for $0 \le x \le c$ is equal to the area of the region for $c \le x \le \pi/2$, the c must be

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{9}$ (E) $\frac{5\pi}{18}$



$\int_0^c \cos x\,dx = \int_c^{\pi/2} \cos x\,dx$

$\sin x \Big|_0^c = \sin x \Big|_c^{\pi/2}$

$\sin(c) - \sin(0) = \sin\frac{\pi}{2} - \sin(c)$

$2\sin(c) = 1$

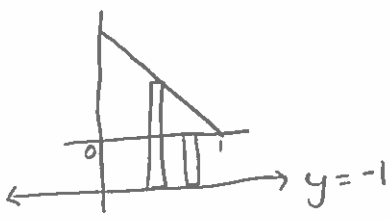
$\sin(c) = \frac{1}{2}$

$c = \arcsin(1/2) \rightarrow c = \frac{\pi}{6}$

8) The region enclosed by the line $x + y = 1$ and the coordinate axes is rotated about the line $y = -1$. What is the volume of the solid generated?

$y = -x + 1$

(A) $\frac{17\pi}{2}$ (B) $\frac{12\pi}{4}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$ (E) $\frac{4\pi}{3}$



$V = \pi \int_0^1 (-x + 1 + 1)^2 dx - \pi \int_0^1 (1)^2 dx$

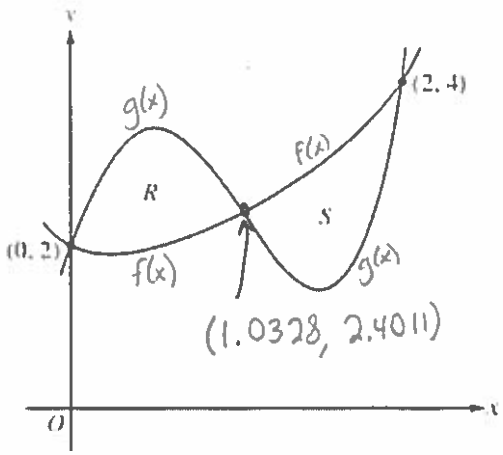
9) 2015 Question #2 Calculator

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.



a) $A = \int_0^{1.0328} (g(x) - f(x))\,dx + \int_{1.0328}^{2} (f(x) - g(x))\,dx$

$A = .997426 + 1.006919$

Area = 2.004

b) $V = \int_{1.0328}^{2} (f(x) - g(x))^2\,dx$

V = 1.283

c) $h = f(x) - g(x)$

$h' = f'(x) - g'(x)$

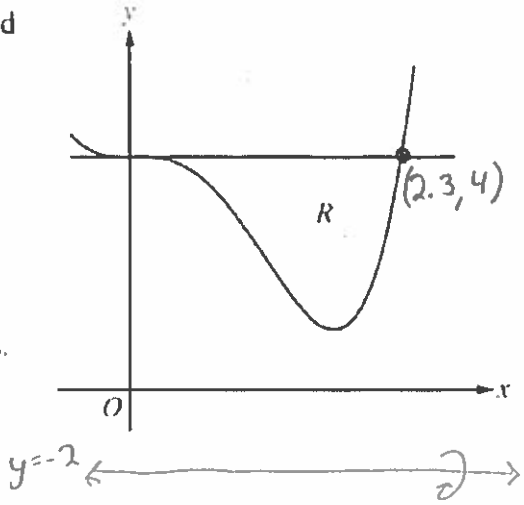
$h' = \left[1 + e^{x^2-2x}(2x-2)\right] - \left[4x^3 - 13x + 6\right]$

$h'(1.8) = 2.11628 - 5.92800$

h'(1.8) = -3.812

10) 2014 Question #2 Calculator

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.



(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

a) $V = \pi \int_0^{2.3} (4+2)^2 dx - \pi \int_0^{2.3} (x^4 - 2.3x^3 + 4 + 2)^2 dx$

$V = 31.471\pi$

$V = 98.868$

b) $V = \frac{1}{2} \int_0^{2.3} (4 - f(x))^2 dx$

$V = 3.574$

c) $\int_0^{k} (4 - f(x)) dx = \int_k^{2.3} (4 - f(x)) dx$