Chapter 8 Practice AP Questions

Name: Answer Key

1) $\int \frac{dx}{2x^2+3x+1} =$ $\quad \frac{A}{2x+1} + \frac{B}{x+1}$ $\quad 1=(2x+1)B + A(x+1)$

Let $x=-1$: $1=-B$, $-1=B$

Let $x=-1/2$: $1=\frac{1}{2}A$, $2=A$

(A) $2\ln\left|\frac{2x+1}{x+1}\right|+C$ (B) $\ln\left|\frac{(2x+1)^2}{x+1}\right|+C$ (C) $\ln\left|\frac{x+1}{2x+1}\right|+C$

(D) $\ln\left|\frac{2x+1}{x+1}\right|+C$ (E) $\ln\left|(x+1)(2x+1)\right|+C$

$2\int \frac{1}{2x+1}dx - \int \frac{1}{x-1}dx$

$u=2x+1$, $du=2dx$, $\frac{1}{2}du=dx$

$\int \frac{1}{u}du - \ln|x-1|$

$\ln|2x+1| - \ln|x-1|$

2) $\int_1^4 \frac{t-2}{(t+1)(t-4)}dt$ is found by using which of the limits below?

(A) $\lim_{x\to 2}\int_x^4 \frac{t-2}{(t+1)(t-4)}dt$ (B) $\lim_{x\to 1^+}\int_x^4 \frac{t-2}{(t+1)(t-4)}dt$ (C) $\lim_{x\to 4^-}\int_x^4 \frac{t-2}{(t+1)(t-4)}dt$

(D) $\lim_{x\to 1}\int_x^4 \frac{t-2}{(t+1)(t-4)}dt$ (E) $\lim_{x\to 4^-}\int_1^x \frac{t-2}{(t+1)(t-4)}dt$

3) In decomposing $\frac{5x-2}{(x-7)(x+4)}$ by the method of partial fractions, one of the fractions obtained is

(A) $\frac{-2}{(x-7)}$ (B) $\frac{2}{(x-7)}$ (C) $\frac{3}{(x-7)}$ (D) $\frac{3}{(x+4)}$ (E) $\frac{5}{(x+4)}$

$\frac{5x-2}{(x-7)(x+4)} = \frac{A}{x-7} + \frac{B}{x+4}$ $\quad 5x-2 = A(x+4) + B(x-7)$

Let $x=7$: $33=11A$, $3=A$

Let $x=-4$: $-22=-11B$, $2=B$

4) Which of the following improper integrals converges?

√ I. $\int_0^\infty e^{-x}dx$ $\quad -e^{-x}\Big|_0^\infty \to -e^{-\infty} - -e^0 \to -0+1$

X II. $\int_0^1 \frac{1}{x^2}dx$ $\quad \lim_{a\to 0^+}\int_a^1 x^{-2}dx \to \left[-\frac{1}{x}\right]_{0^+}^1 \to -\frac{1}{1} - \frac{1}{0^+} \to -1-\infty$

√ III. $\int_0^1 \frac{1}{\sqrt{x}}dx$ $\quad \lim_{a\to 0^+}\int_a^1 x^{-1/2}dx \to \left[2\sqrt{x}\right]_{0^+}^1 \to 2(1) - 2\sqrt{0^+} \to 2-0$

(A) I only (B) III only (C) I and II (D) II and III (E) I and III

5) $\int_1^{\infty} x^{\frac{-5}{4}} dx$ is

$\lim_{b\to\infty} \int_1^b x^{-\frac{5}{4}} dx \rightarrow \lim_{b\to\infty} \left[-4x^{-1/4}\right]_1^{\infty} \rightarrow \frac{-4}{\sqrt[4]{\infty}} - \frac{-4}{\sqrt[4]{1}} = 0+4$

(A) $\frac{5}{4}$ (B) $\frac{1}{4}$ (C) 4 (D) -4 (E) nonexistent

6) Let f be the function defined for $x > 0$, with $f(e) = 2$ and f', the first derivative of f, given by $f'(x) = x^2 \ln x$.

(a) Write an equation for the line tangent to the graph of f at the point $(e, 2)$.

(b) Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.

(c) Use antidifferentiation to find $f(x)$.

a) $f'(e) = e^2 \ln(e)$

$f'(e) = e^2$

$\boxed{y-2 = e^2(x-e)}$

b) $f''(x) = x^2\left(\frac{1}{x}\right) + \ln x\,(2x)$

$f''(x) = x + 2x\ln x$

f is concave up because $f''(x) > 0$ on the interval $(1,3)$

c) $\int x^2 \ln x\, dx$ $u = \ln x$ $v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx$ $dv = x^2 dx$

$\int x^2 \ln x\, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3\left(\frac{1}{x}\right)dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

$f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

$2 = \frac{1}{3}e^3 \ln e - \frac{1}{9}e^3 + C$

$2 = \frac{e^3}{3} - \frac{e^3}{9} + C$

$2 = \frac{2e^3}{9} + C$

$2 - \frac{2e^3}{9} = C$

$\boxed{f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + 2 - \frac{2e^3}{9}}$

7) Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

$@ -1$ $\quad (-1)^2 - \frac{1}{2}(2)$

$1 - 1 = 0$

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right)$. Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

a) $\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\frac{dy}{dx}$

$$\boxed{\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\left(x^2 - \frac{1}{2}y\right)}$$

b) $\frac{dy}{dx} @ (-2,8)$

$(-2)^2 - \frac{1}{2}(8)$

$4 - 4 = 0$

$\frac{d^2y}{dx^2} @ (-2,8)$

$2(-2) - \frac{1}{2}(0)$

-4

$\boxed{\text{f(x) has a local maximum at } (-2,8) \text{ by the } 2^{nd} \text{ Derivative Test } (f'(x)=0 \text{ and } f''(x)<0)}$

c) $\lim_{x\to -1}\left(\frac{g(x)-2}{3(x+1)^2}\right) = \frac{2-2}{3(-1+1)^2} = \frac{0}{0}$

L'Hop

$\lim_{x\to -1} \frac{g'(x)}{6(x+1)} = \frac{\frac{dy}{dx}@(-1)}{6(x+1)} = \frac{0}{0}$

$\frac{dy}{dx} @ -1 \quad (-1)^2 - \frac{1}{2}(2)$

$1 - 1$

0

L'Hop

$\lim_{x\to -1} \frac{g''(x)}{6} = \frac{\frac{d^2y}{dx^2}@(-1)}{6} = \frac{-2}{6} = \boxed{-\frac{1}{3}}$

d) $y_1 = y_0 + \Delta x(f'(x_0, y_0))$

$y_1 = 2 + .5(0 - 1)$

$y_1 = 2 + .5(-1)$

$y_1 = 2 - .5$

$y_1 = 1.5$

$y_2 = 1.5 + .5(.5^2 - \frac{1}{2}(1.5))$

$y_2 = 1.5 + .5(.25 - .75)$

$y_2 = 1.5 + .5(-.5)$

$y_2 = 1.5 - .25$

$y_2 = 1.25$

x	0	.5	1
y	2	1.5	1.25

$\boxed{h(1) \approx 1.25}$