AP Questions Tay/Mac #1 Name: Answer Key

1) Use a 6th degree Maclaurin polynomial to approximate cos(.4)

$$P(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\cos(.4) \approx 1 - \frac{(.4)^2}{2} + \frac{(.4)^4}{4!} - \frac{(.4)^6}{6!} \approx .921$$

2) What are all values of x for which the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n} x^n$ converges?

(A) $\cancel{\text{A}}$ $-e < x \le e$

(B) $-1 \le x < 1$

(C) $\cancel{\text{C}}$ $-e \le x < e$

(D) $-1 < x \le 1$

(E) $-1 \le x \le 1$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{\ln(n+1)} \cdot \frac{\ln(n)}{x^n} \right|$$

L'Hop

$$\lim_{n\to\infty} \left| \frac{x \ln(n)}{\ln(n+1)} \right|$$

$$\lim_{n\to\infty} \left| \frac{x(1/n)}{(1/n+1)} \right| = \left| \frac{x(n+1)}{n} \right| \xrightarrow{\text{L'Hop}} \lim_{n\to\infty} |x| \rightarrow -1 < x < 1$$

let $x=-1$ $\sum_{n=2}^{\infty} \frac{(-1)^n(-1)^n}{\ln n} \rightarrow \sum_{n=2}^{\infty} \frac{1}{\ln n}$ diverges

let $x=1$ $\sum_{n=2}^{\infty} \frac{(-1)^n(1)^n}{\ln n} \rightarrow \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges

3) The Taylor series for $\frac{\sin(x^2)}{x^2}$ centered at x = 0 is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!}$$

$$\frac{\sin(x^2)}{x^2} = 1 - \frac{x^4}{3!} + \frac{x^8}{5!}$$

(A) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$ (B) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k+1)!}$ (C) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k)!}$

(D) $\frac{1}{x} + \sum_{k=1}^{\infty} \frac{(-1)^k x^{2k-1}}{(2k-1)!}$ (E) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{4k}}{(2k+1)!}$

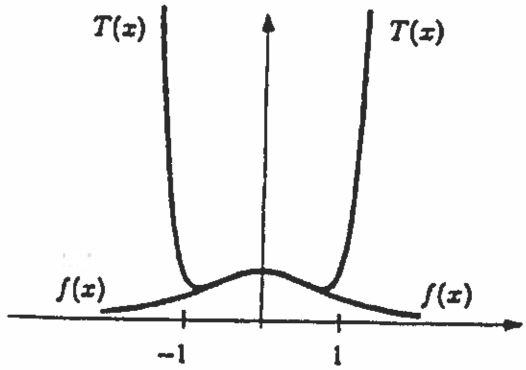
4) The figure to the right shows the graph of y = f(x) and y = T(x) where T(x) is a Taylor polynomial for f(x) centered at zero. Which of the following statements are true?

√ I. T(0.5) is a good approximation for f(0.5)

X II. T(1.5) is a good approximation for f(1.5)

√√ III. T(0) = f(0)

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III



5) The first three nonzero terms in the Taylor series about x = 0 of xe^{-x} are

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}$ $\qquad$ $xe^{-x} = x - x^2 + \frac{x^3}{2}$

$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!}$

(A) $x - x^2 - \frac{x^3}{2!}$ (B) $x + x^2 + \frac{x^3}{2!}$ (C) $-x + x^2 - \frac{x^3}{2!}$ (D) $x - x^2 + \frac{x^3}{2!}$ (E) $1 - x + \frac{x^2}{2!}$

6) For all x if f(x) = $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$, then f'(x) = $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} (2n+1) x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{2n!}$

(A) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n+1)!}$ (B) $\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{2n}}{(2n)!}$ (C) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n+2)!}$

(D) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)!}$ (E) $\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{2n}}{(2n+1)!}$

7) Let E be the error when the Taylor polynomial T(x) = $x - \frac{x^3}{3!}$ is used to approximate f(x) = sinx at x = 0.5. Which of the following is true? Alt series remainder

$\frac{x^5}{5!}$ $\qquad$ $\frac{(.5)^5}{5!} = .00026$

(A) |E| < 0.0001 (B) 0.0001 < |E| < 0.0003 (C) 0.0003 < |E| < 0.005

(D) 0.005 < |E| < 0.007 (E) 0.07 < |E|

(8) The Taylor Series of a function f(x) about x = 3 is given by

$$f(x) = 1 + \frac{3(x-3)}{1!} + \frac{5(x-3)^2}{2!} + \frac{7(x-3)^3}{3!} + \ldots + \frac{(2n+1)(x-3)^n}{n!} + \ldots$$

What is the value of f'''(3)?

$\frac{f'''(3)(x-3)^3}{3!}$

(A) 0
(B) 1.167
(C) 2.5
(D) 5
(E) 7