Calculus AB Review Integrals, Area, FTC Name: Answer Key

1) $\int \frac{1}{x^2}dx =$ $\int x^{-2}dx$ $\frac{x^{-1}}{-1} + C$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

2) $\int(\sin(2x) + \cos(2x))dx =$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$\frac{1}{2}\int(\sin(u) + \cos(u))du$

$\frac{1}{2}(-\cos(u) + \sin(u)) + C$

(A) ½cos(2x) + ½sin(2x) + C

(B) -½cos(2x) + ½sin(2x) + C

(C) 2cos(2x) + 2sin(2x) + C

(D) -2cos(2x) + 2sin(2x) + C

3) The graph of the piecewise linear function f is shown in the figure to the right. If $g(x) = \int_1^x f(t)dt$, evaluate each of the following.

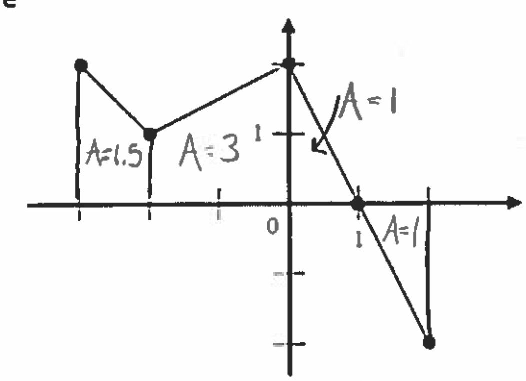
(a) g(-3) = -5.5

(b) g(-2) = -4

(c) g(0) = -1

(d) g(1) = 0

(e) g(2) = -1



Graph of f

4) $\int \frac{x}{x^2-4}dx =$

$u = x^2 - 4$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}du$

$\frac{1}{2}\ln|u| + C$

(A) $\frac{-1}{4(x^2-4)^2} + C$ (B) $\frac{1}{2(x^2-4)} + C$ (C) $\frac{1}{2}\ln|x^2-4| + C$ (D) $2\ln|x^2-4| + C$ (E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

5) If $\int_{-5}^{2} f(x)dx = -17$ and $\int_{5}^{2} f(x)dx = -4$, what is the value of $\int_{-5}^{5} f(x)dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

$\int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx = \int_{-5}^{5} f(x)\,dx$ $-17 + 4 = \int_{-5}^{5} f(x)\,dx$

6) If G(x) is an antiderivative for f(x) and G(2) = -7, then G(4) =

(A) f '(4) (B) -7 + f '(4) (C) $\int_2^4 f(t)dt$ (D) $\int_2^4(-7+f(t))dt$ (E) $-7+\int_2^4 f(t)dt$

$\int_2^4 f(t)dt = G(4)-G(2) \rightarrow G(4) = G(2) + \int_2^4 f(t)dt$

7) What is the average value of $y = \frac{cosx}{x^2+x+2}$ on the closed interval [-1, 3]?

(A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

$\frac{1}{3-^-1}\int_{-1}^{3} \frac{\cos x}{x^2+x+2} dx$

8) $\int x^2cos(x^3)dx =$

$u = x^3$
$du = 3x^2 dx$
$\frac{1}{3}du = x^2 dx$
$\frac{1}{3}\int \cos u \, du$
$\frac{1}{3}\sin u + C$

(A) $-\frac{1}{3}sin(x^3)+C$

(B) $\frac{1}{3}sin(x^3)+C$

(C) $-\frac{x^3}{3}sin(x^3)+C$

(D) $\frac{x^3}{3}sin(x^3)+C$

(E) $\frac{x^3}{3}sin\left(\frac{x^4}{4}\right)+C$

9) $\frac{d}{dx}\left(\int_0^{x^2} sin(t^3)dt\right) =$

$\sin((x^2)^3) * 2x$

$2x \sin x^6$

(A) $-\cos(x^6)$

(B) $\sin(x^3)$

(C) $\sin(x^6)$

(D) $2x\sin(x^3)$

(E) $2x\sin(x^6)$

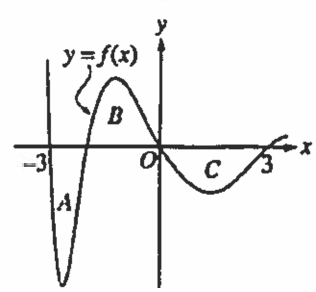
10) Using the substitution u = 2x + 1, $\int_0^2 \sqrt{2x+1}dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2}\sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2\sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5\sqrt{u}\,du$ (D) $\int_0^2\sqrt{u}\,du$ (E) $\int_1^5\sqrt{u}\,du$

$u = 2x+1$
$du = 2dx$
$\frac{1}{2}du = dx$

$u(2) = 5$
$u(0) = 1$

11) The regions A, B, and C in the figure to the right are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3}(f(x)+1)dx$?



$\int_{-3}^{3} f(x)dx + \int_{-3}^{3} 1 dx$

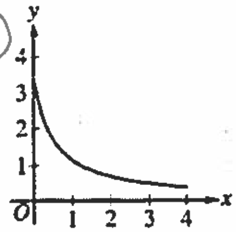
(A) -2 (B) -1 (C) 4 (D) 7 (E) 12

$-2 + x\Big|_{-3}^{3}$

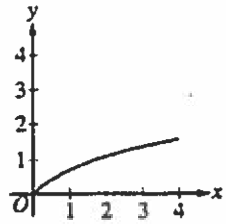
$-2 + (3-(-3))$

12) If a trapezoidal sum over approximates $\int_0^4 f(x)dx$, and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of y = f(x)?

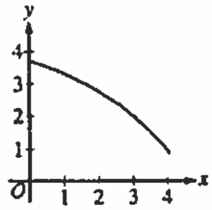
(A)



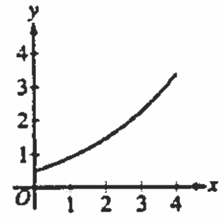
(B)



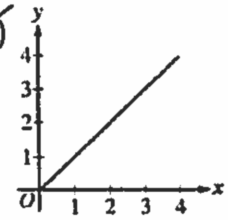
~~(C)~~



(D)



~~(E)~~



13) There is no snow of Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{cost}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. (t = 6). The rate g(t), in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & for\ 0 \le t < 6 \\ 125 & for\ 6 \le t \le 7 \\ 108 & for\ 7 \le t \le 9 \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

$\int_0^6 F(t)dt = \boxed{142.275\ ft^3}$

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

$F(8) - 108$

$48.417 - 108$

$\boxed{-59.583\ ft^3/hr}$

(c) Let h(t) represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain 0 ≤ t ≤ 9.

$$h(t) = \begin{cases} 0, & 0 \le t \le 6 \\ 125(t-6), & 6 \le t \le 7 \\ 108(t-7)+125, & 7 \le t \le 8 \end{cases}$$

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

$\int_0^9 F(t)dt - 125 - 216$

$367.335 - 125 - 216$

$\boxed{26.335\ ft^3}$

14)

Distance x (mm)	0	60	120	180	240	300	360
Diameter B(x) (mm)	24	30	28	30	26	24	26

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and B(x) is a twice-differentiable function that represents the diameter at that point.

(a) Write an integral expression in terms of B(x) that represents the average radius, in mm, of the blood vessel between x = 0 and x = 360.

$$\frac{1}{360-0}\int_0^{360} \frac{1}{2}B(x)\,dx$$

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

$$\frac{1}{2}\cdot\frac{1}{360}\left[120(30) + 120(30) + 120(24)\right]$$

$$\boxed{14\text{ mm}}$$

(c) Using correct units, explain the meaning of $\pi \int_{125}^{175} \left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

$$\pi \int_a^b r^2\,dx$$

This is the volume of the blood vessel between 125 mm and 175 mm distance from the end of the blood vessel measured in cubic mm.

15) The velocity of a particle moving along the x-axis is given by the expression $v(t) = 3t - t^3$. The particle has the position x = 5 when t = 2. Write an equation for x(t) the position of the particle and use it to determine the position of the particle when t = 1.

$v(t) = 3t - t^3$

$x(t) = \frac{3}{2}t^2 - \frac{1}{4}t^4 + C$

$5 = \frac{3}{2}(2)^2 - \frac{1}{4}(2)^4 + C$

$5 = 6 - 4 + C$

$3 = C$

$x(1) = \frac{3}{2}(1)^2 - \frac{1}{4}(1)^4 + 3$

$x(1) = 1.5 - .25 + 3$

$$\boxed{x(t) = \frac{3}{2}t^2 - \frac{1}{4}t^4 + 3 \qquad x(1) = 4.25}$$