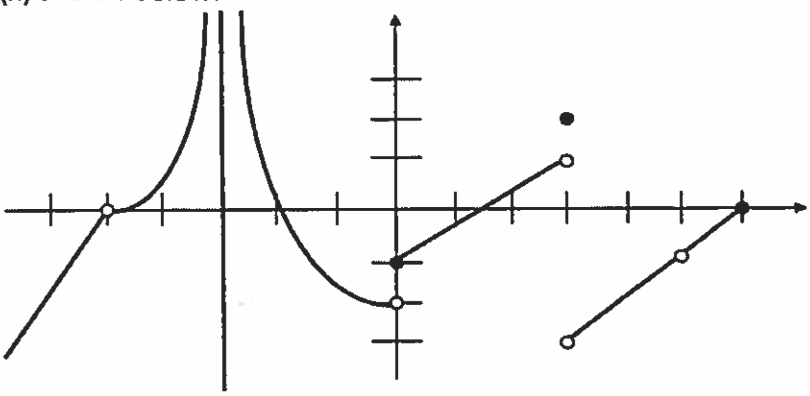
Calculus AB Review Limits and Derivatives Name: Answer Key

1) Answer the following using the graph of f(x) shown below.



(a) f(0) = -1

(b) f(3) = 2

(c) $\lim_{x\to-5} f(x)$ = 0

(d) $\lim_{x\to0^+} f(x)$ = -1

(e) $\lim_{x\to3^-} f(x)$ = 1

2) Let $f(x)=\begin{cases}3x^2+1, x<1\\ 4x, \quad x\geq 1\end{cases}$. Which of the following is true?

✓ I. f(x) is continuous at x = 1

X II. f(x) is differentiable at x = 1

✓ III. $\lim_{x\to1^-} f(x)=\lim_{x\to1^+} f(x)$

Continuous

$\lim_{x\to1^-} f(x)=\lim_{x\to1^+} f(x)$

$3(1)^2+1=4(1)$

$4=4$ ✓

differentiable

$\lim_{x\to1^-} f'(x)=\lim_{x\to1^+} f'(x)$

$6(1)=4$

$6=4$ X

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

3) $\lim_{h\to0}\frac{\cos(\frac{\pi}{6}+h)-\cos(\frac{\pi}{6})}{h}=$

$\frac{d}{dx}[\cos x]=-\sin x$ $-\sin(\frac{\pi}{6})\to-\frac{1}{2}$

(A) Does not exist (B) 1/2 (C) -1/2 (D) $\sqrt{3}/2$ (E) $-\sqrt{3}/2$

4) Find the value of the limit: $\lim_{h\to0}\frac{\sqrt{\tan(2x+2h)}-\sqrt{\tan(2x)}}{h}$

$\frac{d}{dx}\left[\sqrt{\tan(2x)}\right]\to\frac{d}{dx}(\tan(2x))^{1/2}$

$\frac{1}{2}(\tan(2x))^{-1/2}*\sec^2(2x)*(2)$

$\boxed{\frac{\sec^2(2x)}{\sqrt{\tan(2x)}}}$

5) Let *f* be a differentiable function with *f*(2) = 3 and *f* '(2) = -5, and let *g* be the function defined by $g(x)=x\cdot f(x)$. What is the equation for the line tangent to the graph of *g* at the point where x = 2?

$g'(x)=x f'(x)+f(x)(1)$

$g'(x)=x f'(x)+f(x)$

$g'(2)=2f'(2)+f(2)=2(-5)+3=-7$

$g(2)=2f(2)$

$g(2)=2(3)=6$

$\boxed{y-6=-7(x-2)}$

Find the derivatives of the following functions.

6) $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

$f'(x) = (3x^2+7)(2x-2) + (x^2-2x+3)(6x)$

7) $f(x) = \sqrt{x} \cdot \sin x$

$f'(x) = x^{1/2} \cos x + \sin x \left(\frac{1}{2}x^{-1/2}\right)$

$f'(x) = \sqrt{x} \cos x + \dfrac{\sin x}{2\sqrt{x}}$

8) $f(x) = 3x^2 \sec^3 x \rightarrow (3x^2)(\sec x)^3$

$f'(x) = 3x^2\left(3(\sec x)^2 (\sec x \tan x)\right) + \sec^3 x\,(6x)$

$f'(x) = 9x^2 \sec^3 x \tan x + 6x \sec^3 x$

9) $f(x) = \dfrac{x^4+x}{\tan^2 x} \rightarrow \dfrac{x^4+x}{(\tan x)^2}$

$f'(x) = \dfrac{\tan^2 x\,(4x^3+1) - (x^4+x)(2(\tan x) * (\sec^2 x))}{(\tan x)^4}$

10) Given the equation $y = \sin(3x + 4y)$, find $\frac{dy}{dx}$.

$\frac{dy}{dx} = \cos(3x+4y) * \left(3 + 4\frac{dy}{dx}\right)$

$\frac{dy}{dx} = 3\cos(3x+4y) + 4\cos(3x+4y)\frac{dy}{dx}$

$\frac{dy}{dx} - 4\cos(3x+4y)\frac{dy}{dx} = 3\cos(3x+4y)$

$\frac{dy}{dx}\left(1 - 4\cos(3x+4y)\right) = 3\cos(3x+4y)$

$\frac{dy}{dx} = \dfrac{3\cos(3x+4y)}{1-4\cos(3x+4y)}$

11) Suppose that *f* and *g* are twice differentiable functions having selected values given in the table below.

x	*f(x)*	*f'(x)*	*g(x)*	*g'(x)*
1	5	4	2	7
2	8	6	-6	-4

If *h(x) = f(g(x))*, what is the value of *h'(x)* at the point where x = 1?

$h'(x) = f'(g(x)) * g'(x)$

$h'(1) = f'(g(1)) * g'(1)$

$\rightarrow h'(1) = f'(2) * 7$

$h'(1) = 6 * 7$

$\rightarrow h'(1) = 42$

12) A particle moves along the x-axis according to the position function $x(t) = 3\sin(2t) + 1$.

(a) Determine the instantaneous velocity of the particle at $t = \pi$. Which direction is the particle moving?

$v(t) = 3\cos(2t) * 2$ $\qquad$ $v(\pi) = 6\cos(2\pi)$

$v(t) = 6\cos(2t)$ $\qquad$ $v(\pi) = 6$ ← moving right (positive)

(b) What is the acceleration of the particle at $t = \frac{\pi}{4}$?

$a(t) = -6\sin(2t) * 2$ $\qquad$ $a\left(\frac{\pi}{4}\right) = -12\sin\left(\frac{\pi}{2}\right)$

$a(t) = -12\sin(2t)$ $\qquad$ $a\left(\frac{\pi}{4}\right) = -12$

(c) Is the particle speeding up or slowing down at $t = \frac{\pi}{4}$? Justify your answer.

$v\left(\frac{\pi}{4}\right) = 6\cos\left(\frac{\pi}{2}\right)$ $\qquad$ The particle is at rest so it is neither

$v\left(\frac{\pi}{4}\right) = 6(0) = 0$ $\qquad$ speeding up nor slowing down

13) If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(14)}$ is the same as

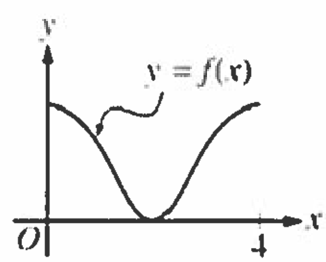
(A) y $\qquad$ $y = -\sin x$ $\qquad$ 4 8 12

(B) $\frac{dy}{dx}$ $\qquad$ $y' = -\cos x$ $\qquad$ 5 9 13

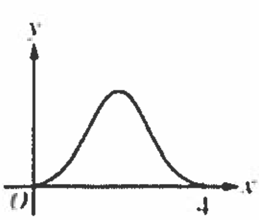
(C) $\frac{d^2y}{dx^2}$ $\qquad$ $y'' = \sin x$ $\qquad$ 6 10 14

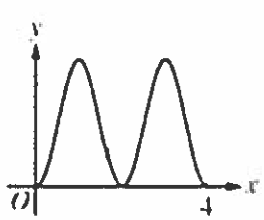
(D) $\frac{d^3y}{dx^3}$ $\qquad$ $y''' = \cos x$ $\qquad$ 7 11

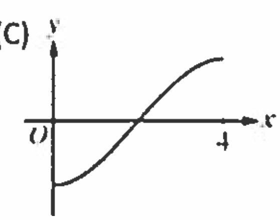
14)

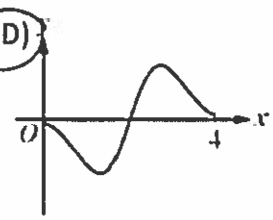


The graph of $y = f(x)$ on the closed interval [0, 4] is shown above. Which of the following could be the graph of $y = f'(x)$?

(A)


(B)


(C)


(D)


15)

t (hours)	0	1	3	4	7	8	9
L(t) (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of L(t) at various times t are shown in the table above.

(a) Use the data in the tale to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. (t = 5.5). Show the computations that lead to your answer. Indicate units of measure.

$$\frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{7 - 4} = \frac{24}{3} = 8 \text{ people/hr}$$

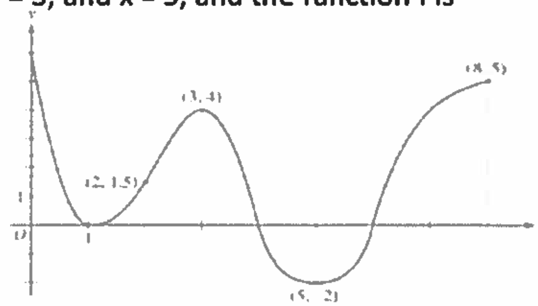
(b) For $0 \le t \le 9$, what is the fewest number of times at which L'(t) must equal 0? Give a reason for your answer. 3 times is the fewest. $L'(t) = 0$ when switching from increasing to decreasing or vice versa (between 3 and 4, 4 and 7, 7 and 8)

(c) Is there a time on the interval [1, ~~9~~ 4] where the rate at which the number of people waiting in line was decreasing at ~~6~~ 10 people per hour? Justify your answer.

$$\frac{L(4) - L(1)}{4 - 1} = \frac{126 - 156}{4 - 1} = \frac{-30}{3} = -10 \text{ people/hr}$$

By the Mean Value Theorem, there must exist a time on the interval [1,4] where the rate is -10 people/hr.

16) The figure below shows the graph of f ', the derivative of a twice differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f ' has horizontal tangent lines at x = 1, x = 3, and x = 5, and the function f is defined for all real numbers.



Graph of f'

(a) Find all values of x on the open interval 0 < x < 8 for which the function *f* has a local maximum. Justify your answer.

X=4 The slope switches from pos. to neg.

(b) On what open intervals contained in 0 < x < 8 is the graph of *f* both concave down and increasing? Explain your reasoning. (0,1) and (3,4) The slope is positive and decreasing

(c) Does the tangent line to the graph of y = f(x) at the point where x = 4 lie above or below the curve near that point? Justify your response. The graph of y is concave down at x=4, so the tangent line is above the graph.