Practice Integration Problems #1

Name: ______________________________

1) A faucet was turned on at t=0, and t minutes later water was flowing into a barrel at a rate of $t^2 + 4t$ gallons per minute, $0 \le t \le 5$.

a) How much water was added to the barrel during these 5 minutes?

b) Find the average flow rate for these five minutes.

2) (from 2000 Free Response) Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$. At $t = 0$, the tank contains 30 gallons of water.

a) How many gallons of water leak out of the tank from $t = 0$ to $t = 3$ minutes?

b) How many gallons of water are in the tank at time $t = 3$ minutes?

c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

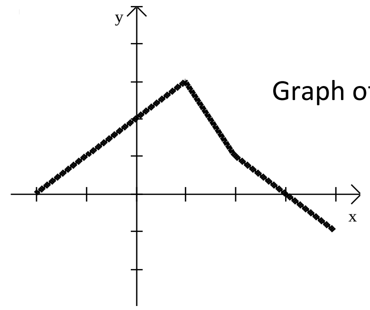
d) At what time t, for $0 \le t \le 120$, is the water in the tank a maximum? Justify your answer.

3) If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)dt$ represent?

4) A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

5) The graph of the function f, consisting of three line segments, is shown on the right.

Let $g(x)=\int_1^x f(t)\,dt$.



Graph of f

(a) Find $g(2),\ g(4),\ g(-2)$.

(b) Find $g'(0)$ and $g'(3)$.

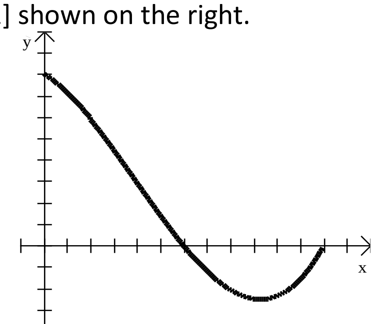
(c) Find the instantaneous rate of change of g with respect to x at $x = 2$.

(d) Find the absolute maximum value of g on the interval $[-2,\ 4]$. Justify your answer.

(e) The second derivative of g is not defined at $x = 1$ and at $x = 2$. Which of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

6) Let $H(x)=\int_0^x f(t)\,dt$ where f is the continuous function with domain [0, 12] shown on the right.

(a) Find $H(0)$.



Graph of f

(b) On what interval(s) of x is H increasing? Justify your answer.

(c) On what interval(s) of x is H concave up? Justify your answer.

(d) Is $H(12)$ positive or negative? Explain.

(e) For what value of x does H achieve its maximum value? Explain.

7) If $f(1)=12$, f' is continuous, and $\int_1^4 f'(x)\,dx = 17$, what is the value of $f(4)$?

8) If $\int_2^5 (2f(x)+3)\,dx = 17$, find $\int_2^5 f(x)\,dx$.

9) Consider the function f that is continuous on the interval $[-5,5]$ and for which $\int_0^5 f(x)\,dx = 4$. Evaluate:

(a) $\int_0^5 (f(x)+2)\,dx =$

(c) $\int_{-5}^5 f(x)\,dx$ (f is even) $=$

(b) $\int_{-2}^3 f(x+2)\,dx =$

(d) $\int_{-5}^5 f(x)\,dx$ (f is odd) $=$

10) A bowl of soup is placed on the kitchen counter to cool. The temperature of the soup is given in the table below.

(a) Find $\int_0^{12} T'(x)\,dx$.

Time *t* (minutes)	0	5	8	12
Temperature $T(x)$ (°F)	105	99	97	93

(b) Find the average rate of change of $T(x)$ over the time interval *t* = 5 to *t* = 8 minutes.

11) If f and g are continuous functions such that $g'(x) = f(x)$ for all x, then $\int_2^3 f(x)\,dx =$

(A) $g'(2) - g'(3)$ (B) $g'(3) - g'(2)$ (C) $g(3) - g(2)$

(D) $f(3) - f(2)$ (E) $f'(3) - f'(2)$