Practice Integration Problems #1 Name: Answer Key

1) A faucet was turned on at t=0, and t minutes later water was flowing into a barrel at a rate of t^2+4t gallons per minute, $0 \le t \le 5$.

a) How much water was added to the barrel during these 5 minutes?

$$\int_0^5 (t^2+4t)\,dt \qquad \frac{1}{3}t^3+2t^2\Big|_0^5 = \frac{125}{3}+50 = 91.\bar{6} \text{ gallons}$$

b) Find the average flow rate for these five minutes.

$$\frac{1}{5-0}\int_0^5 (t^2+4t)\,dt = \frac{1}{5}(91.\bar{6}) = 18.\bar{3} \text{ gal/min}$$

2) (from 2000 Free Response) Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$. At $t=0$, the tank contains 30 gallons of water.

a) How many gallons of water leak out of the tank from $t=0$ to $t=3$ minutes?

$$\int_0^3 \sqrt{t+1}\,dt = 4.\bar{6} \text{ gallons}$$

b) How many gallons of water are in the tank at time $t=3$ minutes?

$$30+\int_0^3 8\,dt - \int_0^3 \sqrt{t+1}\,dt = 30+24-4.\bar{6} = 49.\bar{3} \text{ gallons}$$

c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

$$A(t) = 30+\int_0^t 8\,dx - \int_0^t \sqrt{x+1}\,dx$$

d) At what time t, for $0 \le t \le 120$, is the water in the tank a maximum? Justify your answer.

$8=\sqrt{t+1}$

$64=t+1$

$t=63$ mins.

At 63 minutes the water going in the tank equals the water going out. Before 63 min, more water entered than left.

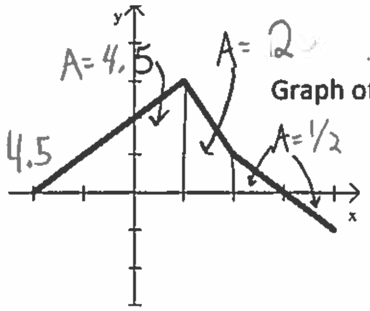
3) If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)dt$ represent?

The integral represents the total amount of weight gained between $t=5$ and $t=10$

4) A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100+\int_0^{15} n'(t)dt$ represent? The integral represents the total number of bees after 15 weeks.

5) The graph of the function f, consisting of three line segments, is shown on the right.

Let $g(x)=\int_1^x f(t)\,dt$.



Graph of f

(a) Find $g(2)$, $g(4)$, $g(-2)$.

$g(2)=\int_1^2 f(x)dx=2$ $\quad g(4)=\int_1^4 f(x)dx=2$ $\quad g(-2)=\int_1^{-2} f(x)dx=4.5$

(b) Find $g'(0)$ and $g'(3)$.

$g'(x)=f(x)$ $\quad g'(0)=f(0)=2$ $\quad g'(3)=f(3)=0$

(c) Find the instantaneous rate of change of g with respect to x at $x = 2$.

$g'(2)=f(2)=1$

(d) Find the absolute maximum value of g on the interval $[-2, 4]$. Justify your answer.

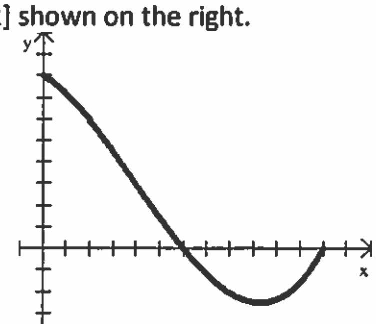
$g(3)=\int_1^3 f(x)dx=2.5$

Absolute extrema occur at critical points or endpoints. $g(3)=2.5$ $\quad g(2)=g(4)=2$ $\quad g(1)=0$ $\quad g(-2)=-4.5$

(e) The second derivative of g is not defined at $x = 1$ and at $x = 2$. Which of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

$x=1$ is a point of inflection because $g''(x)=f'(x)$ changes signs at this point

6) Let $H(x)=\int_0^x f(t)\,dt$ where f is the continuous function with domain [0, 12] shown on the right.


Graph of f

(a) Find $H(0)$. $\quad H(0)=\int_0^0 f(t)dt=0$

(b) On what interval(s) of x is H increasing? Justify your answer.

H is increasing on $(0,6)$ b/c $f(t)$ is positive on this interval.

(c) On what interval(s) of x is H concave up? Justify your answer.

$H''(x)=f'(x)$ $\quad$ H is concave up on $(9,12)$ b/c the slope of $f(x)$ is positive.

(d) Is $H(12)$ positive or negative? Explain.

$H(12)$ is positive b/c $\int_0^6 f(t)dt > \int_6^{12} f(t)dt$

(e) For what value of x does H achieve its maximum value? Explain.

The maximum value of H is at $x=6$ b/c the graph as a relative max at that point and $H(6)>H(12)>H(0)$

7) If $f(1)=12$, f' is continuous, and $\int_1^4 f'(x)\,dx=17$, what is the value of $f(4)$?

F

$\int_1^4 f'(x)dx = f(4) - f(1)$

$17 = f(4) - (12)$

$f(4) = 29$

8) If $\int_2^5 (2f(x)+3)\,dx=17$, find $\int_2^5 f(x)\,dx$.

$2\int_2^5 f(x)\,dx + \int_2^5 3dx = 17$

$2\int_2^5 f(x)dx + 9 = 17$

$2\int_2^5 f(x)dx = 8$

$\int_2^5 f(x)dx = 4$

9) Consider the function f that is continuous on the interval $[-5,5]$ and for which $\int_0^5 f(x)\,dx=4$. Evaluate:

(a) $\int_0^5 (f(x)+2)\,dx = 4 + \int_0^5 2dx$

$4 + 10$

14

(b) $\int_{-2}^3 f(x+2)\,dx =$

$F(5) - F(0) = \int_0^5 f(x)$

4

(c) $\int_{-5}^5 f(x)\,dx$ (f is even) $=$

$4 \times 2 = 8$

(d) $\int_{-5}^5 f(x)\,dx$ (f is odd) $=$

$-4 + 4$

0

10) A bowl of soup is placed on the kitchen counter to cool. The temperature of the soup is given in the table below.

Time t (minutes)	0	5	8	12
Temperature $T(x)$ (°F)	105	99	97	93

(a) Find $\int_0^{12} T'(x)\,dx$.

$T(12) - T(0)$

$93 - 105 = -12$

(b) Find the average rate of change of $T(x)$ over the time interval $t = 5$ to $t = 8$ minutes.

$\frac{T(8) - T(5)}{8-5} = \frac{97-99}{3} = -\frac{2}{3}$ °F/min

11) If f and g are continuous functions such that $g'(x)=f(x)$ for all x, then $\int_2^3 f(x)\,dx = \int_2^3 g'(x)dx$

(A) $g'(2)-g'(3)$ (B) $g'(3)-g'(2)$ (C) $g(3)-g(2)$

(D) $f(3)-f(2)$ (E) $f'(3)-f'(2)$