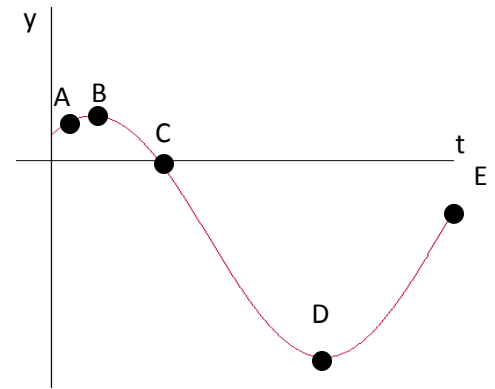


## Practice Integration Problems #2

Name: \_\_\_\_\_

1) The figure to the right shows the graph of the velocity of a particle moving along the x-axis as a function of time. If the particle is at the origin when  $t = 0$ , then which of the marked points is the particle furthest from the origin?



- (A) A                      (B) B                      (C) C  
 (D) D                      (E) E

2)  $\int \sin(3x+4)dx =$

- (A)  $-\frac{1}{3}\cos(3x+4)+C$               (B)  $-\cos(3x+4)+C$               (C)  $-3\cos(3x+4)+C$   
 (D)  $\cos(3x+4)+C$               (E)  $\frac{1}{3}\cos(3x+4)+C$

3) Let  $f(x)$  be the function defined by  $f(x) = \begin{cases} x, & x \leq 0 \\ x+1, & x > 0 \end{cases}$ . The value of  $\int_{-2}^1 xf'(x)dx =$

- (A)  $3/2$   
 (B)  $5/2$   
 (C)  $3$   
 (D)  $7/2$   
 (E)  $11/2$

4) The average value of the function  $f(x) = \cos\left(\frac{1}{2}x\right)$  on the closed interval  $[-4, 0]$  is

- (A)  $-1/2\sin(2)$               (B)  $-1/4\sin(2)$               (C)  $1/2\cos(2)$               (D)  $1/4\sin(2)$               (E)  $1/2\sin(2)$

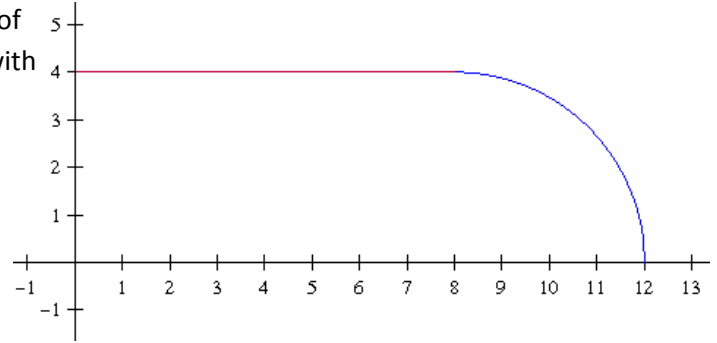
5) Let  $R(t)$  represent the rate at which water is leaking out of a tank, where  $t$  is measured in hours. Which of the following expressions represents the total amount of water in gallons that leaks out in the first three hours?

- (A)  $R(3) - R(0)$       (B)  $\frac{R(3) - R(0)}{3 - 0}$       (C)  $\int_0^3 R(t) dt$       (D)  $\int_0^3 R'(t) dt$       (E)  $\frac{1}{3} \int_0^3 R(t) dt$

6) Suppose that  $f(x)$  is an even function and let  $\int_0^1 f(x) dx = 5$  and  $\int_0^7 f(x) dx = 1$ . What is  $\int_{-7}^{-1} f(x) dx$ ?

- (A) -5  
(B) -4  
(C) 0  
(D) 4  
(E) 5

7) As shown in the figure to the right, the function  $f(x)$  consists of a line segment from  $(0, 4)$  to  $(8, 4)$  and one-quarter of a circle with a radius of 4. What is the average (mean) value of this function on the interval  $[0, 12]$ ? (calc.)



- (A) 2  
(B) 3.714  
(C) 3.9  
(D) 22.283  
(E) 41.144

8) If  $f$  is the function defined by  $f(x) = \sqrt[3]{x^2 + 4x}$  and  $g$  is an antiderivative of  $f$  such that  $g(5) = 7$ , then  $g(1) \approx$  (calc.)

- (A) -3.882  
(B) -3.557  
(C) 1.710  
(D) 3.557  
(E) 3.882

9) If  $f$  and  $g$  are continuously differentiable functions defined for all real numbers, which of the following definite integrals is equal to  $f(g(4)) - f(g(2))$ ?

- (A)  $\int_2^4 f'(g(x)) dx$       (B)  $\int_2^4 f(g(x)) f'(x) dx$       (C)  $\int_2^4 f(g(x)) g'(x) dx$   
(D)  $\int_2^4 f'(g(x)) g'(x) dx$       (E)  $\int_2^4 f'(g'(x)) g'(x) dx$

10) If the substitution  $u = \sqrt{x-1}$  is made, the integral  $\int_2^5 \frac{\sqrt{x-1}}{x} dx =$

(A)  $\int_2^5 \frac{2u^2}{u^2+1} du$

(B)  $\int_1^2 \frac{u^2}{u^2+1} du$

(C)  $\int_1^2 \frac{u^2}{2(u^2+1)} du$

(D)  $\int_2^5 \frac{u}{u^2+1} du$

(E)  $\int_1^2 \frac{2u^2}{u^2+1} du$

11) If  $\int_0^2 (2x^3 - kx^2 + 2k) dx = 12$ , then  $k$  must be

- (A) -3
- (B) -2
- (C) 1
- (D) 2
- (E) 3

12)  $\frac{d}{dx} \int_x^{x^3} \sin(t^2) dt =$

(A)  $\sin(x^6) - \sin(x^2)$

(B)  $6x^2 \sin(x^3) - 2 \sin(x)$

(C)  $3x^2 \sin(x^6) - \sin(x^2)$

(D)  $6x^5 \sin(x^6) - 2 \sin(x^2)$

(E)  $2x^3 \cos(x^6) - 2 \cos(x^2)$

2013 Free Response Question 1 (calc.)

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by  $G(t) = 90 + 45 \cos\left(\frac{t^2}{18}\right)$ , where  $t$  is measured in hours and  $0 \leq t \leq 8$ . At the beginning of the workday ( $t = 0$ ), the plant has 500 tons of unprocessed gravel. During the hours of operation,  $0 \leq t \leq 8$ , the plant processes gravel at a constant rate of 100 tons per hour.

- (a) Find  $G'(5)$ . Using correct units, interpret your answer in the context of the problem.
- (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time  $t = 5$  hours? Show the work that leads to your answer.
- (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.