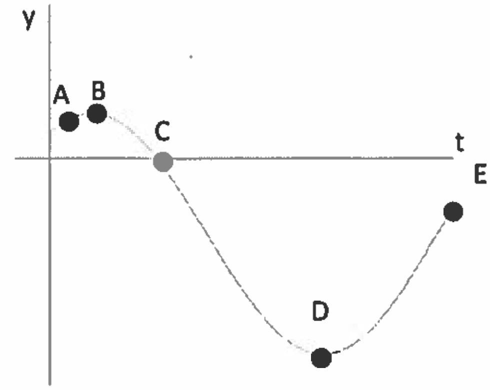
Practice Integration Problems #2 Name: Answer Key

1) The figure to the right shows the graph of the velocity of a particle moving along the x-axis as a function of time. If the particle is at the origin when t = 0, then which of the marked points is the particle furthest from the origin?



(A) A (B) B (C) C

(D) D (E) E

2) $\int \sin(3x+4)dx =$

$u=3x+4$
$du=3dx$
$\frac{1}{3}du=dx$

$\frac{1}{3}\int \sin(u)du$
$-\frac{1}{3}\cos(u)$

(A) $-\frac{1}{3}\cos(3x+4)+C$ (B) $-\cos(3x+4)+C$ (C) $-3\cos(3x+4)+C$

(D) $\cos(3x+4)+C$ (E) $\frac{1}{3}\cos(3x+4)+C$

3) Let f(x) be the function defined by f(x) = $\begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$. The value of $\int_{-2}^{1} xf(x)dx =$

(A) 3/2
(B) 5/2
(C) 3
(D) 7/2
(E) 11/2

$\int_{-2}^{0} x(x)dx + \int_{0}^{1} x(x+1)dx$

$\int_{-2}^{0} x^2 dx + \int_{0}^{1}(x^2+x)dx$

$\frac{1}{3}x^3\Big|_{-2}^{0} + \frac{1}{3}x^3+\frac{1}{2}x^2\Big|_{0}^{1}$

$\frac{1}{3}(0)^3 - \frac{1}{3}(-2)^3 + \left(\frac{1}{3}(1)^3+\frac{1}{2}(1)^2\right) - (0)$

$\frac{8}{3} + \frac{1}{3} + \frac{1}{2}$

$3 + 1/2$

3.5

4) The average value of the function f(x) = $\cos\left(\frac{1}{2}x\right)$ on the closed interval [-4, 0] is

$\sin(-a) \to -\sin(a)$

(A) -1/2sin(2) (B) -1/4sin(2) (C) 1/2cos(2) (D) 1/4sin(2) (E) 1/2sin(2)

$\frac{1}{0--4}\int_{-4}^{0}\cos\left(\frac{1}{2}x\right)$

$u=\frac{1}{2}x$
$du=\frac{1}{2}dx$
$2du=dx$
$u(0)=0$
$u(-4)=-2$

$2\left(\frac{1}{4}\right)\int_{-2}^{0}\cos(u)du$

$\frac{1}{2}\sin(u)\Big|_{-2}^{0}$

$\frac{1}{2}\sin(0) - \frac{1}{2}\sin(-2)$

$0 - \frac{1}{2}\sin(-2)$

$-\frac{1}{2}\sin(-2) \to \frac{1}{2}\sin(2)$

5) Let R(t) represent the rate at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total amount of water in gallons that leaks out in the first three hours?

(A) R(3) – R(0) (B) $\frac{R(3)-R(0)}{3-0}$ (C) $\int_0^3 R(t)dt$ (D) $\int_0^3 R'(t)dt$ (E) $\frac{1}{3}\int_0^3 R(t)dt$

6) Suppose that f(x) is an even function and let $\int_0^1 f(x)dx = 5$ and $\int_0^7 f(x)dx = 1$. What is $\int_{-7}^{-1} f(x)dx$?

(A) – 5
(B) -4
(C) 0
(D) 4
(E) 5

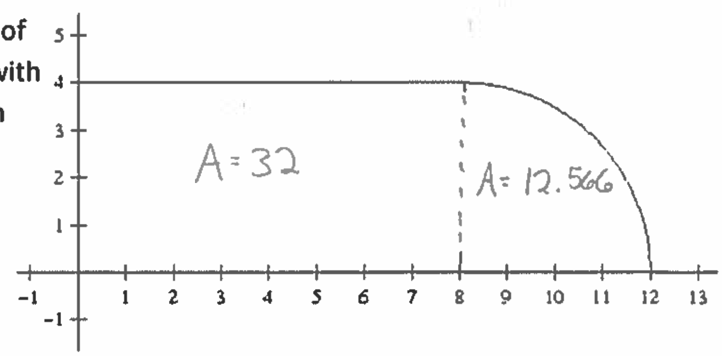
$\int_0^7 f(x)dx = \int_0^1 f(x)dx + \int_1^7 f(x)dx$

$1 = 5 + \int_1^7 f(x)dx$

$-4 = \int_1^7 f(x)dx$

$\int_1^7 f(x)dx = \int_{-7}^{-1} f(x)dx$

7) As shown in the figure to the right, the function f(x) consists of a line segment from (0, 4) to (8, 4) and one-quarter of a circle with a radius of 4. What is the average (mean) value of this function on the interval [0, 12]? (calc.)



(A) 2
(B) 3.714
(C) 3.9
(D) 22.283
(E) 41.144

$\frac{1}{12-0}\int_0^{12} f(x)dx$

$\frac{1}{12}(44.566)$

3.714

8) If f is the function defined by f(x) = $\sqrt[3]{x^2+4x}$ and g is an antiderivative of f such that g(5) = 7, then g(1) ≈ (calc.)

(A) −3.882
(B) −3.557
(C) 1.710
(D) 3.557
(E) 3.882

$\int_1^5 \sqrt[3]{x^2+4x}\,dx = g(5) - g(1)$

$10.882 = 7 - g(1)$

$g(1) = 7 - 10.882$

9) If f and g are continuously differentiable functions defined for all real numbers, which of the following definite integrals is equal to f(g(4)) – f(g(2))?

(A) $\int_2^4 f'(g(x))dx$ (B) $\int_2^4 f(g(x))f'(x)dx$ (C) $\int_2^4 f(g(x))g'(x)dx$

(D) $\int_2^4 f'(g(x))g'(x)dx$ (E) $\int_2^4 f'(g'(x))g'(x)dx$

$u(2)=1 \quad u(5)=2$

$\frac{u}{u^2+1}(2u\,du) \rightarrow \frac{2u^2}{u^2+1}$

10) If the substitution u = $\sqrt{x-1}$ is made, the integral $\int_2^5 \frac{\sqrt{x-1}}{x}dx =$

$u=\sqrt{x-1}$

$u^2 = x-1$

$u^2+1=x$

(A) $\int_2^5 \frac{2u^2}{u^2+1}du$ (B) $\int_1^2 \frac{u^2}{u^2+1}du$ (C) $\int_1^2 \frac{u^2}{2(u^2+1)}du$

(D) $\int_2^5 \frac{u}{u^2+1}du$ (E) $\int_1^2 \frac{2u^2}{u^2+1}du$

$u=(x-1)^{1/2}$

$du = \frac{1}{2}(x-1)^{-1/2}dx$

$du = \frac{1}{2\sqrt{x-1}}dx = \frac{1}{2\sqrt{u^2}}dx = \frac{1}{2u}dx$

$du = \frac{1}{2u}dx$

$2u\,du = dx$

11) If $\int_0^2 (2x^3 - kx^2 + 2k)dx = 12$, then k must be

(A) -3
(B) -2
(C) 1
(D) 2
(E) 3

$\frac{1}{2}x^4 - \frac{k}{3}x^3 + 2kx \Big|_0^2$

$(8 - \frac{8}{3}k + 4k) - (0)$

$8 + \frac{4}{3}k$

$8 + \frac{4}{3}k = 12$

$\frac{4}{3}k = 4$

$k = 4(\frac{3}{4})$

$k=3$

12) $\frac{d}{dx}\int_x^{x^3} \sin(t^2)dt =$

(A) $\sin(x^6) - \sin(x^2)$ (B) $6x^2\sin(x^3) - 2\sin(x)$ (C) $3x^2\sin(x^6) - \sin(x^2)$

(D) $6x^5\sin(x^6) - 2\sin(x^2)$ (E) $2x^3\cos(x^6) - 2\cos(x^2)$

$\int_x^a \sin(t^2)dt + \int_a^{x^3} \sin(t^2)dt$

$-\int_a^x \sin(t^2)dt + \int_a^{x^3} \sin(t^2)dt$

$-\sin(x^2) + \sin(x^6)\cdot 3x^2$

2013 Free Response Question 1 (calc.)

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

a) $G'(t) = -45\sin\left(\frac{t^2}{18}\right) \cdot \frac{2t}{18}$

This is the rate of change for the rate at which unprocessed gravel arrives at the plant.

$G'(5) = -24.588$ tons/hr^2

b) $\int_0^8 G(t)\,dt = 825.551$ tons of gravel

c) $G(5) = 98.141$ ← amount coming in

-100 ← amount being processed

$98.181 - 100 = -1.859$

The amount of unprocessed gravel is decreasing at $t = 5$.

d) $G(t) = 100$ when $t = 4.923$

$$\underset{\text{initial amount}}{500} + \underset{\text{coming in}}{\int_0^{4.9234803} G(t)\,dt} - \underset{\text{going out}}{\int_0^{4.9234803} 100\,dt} = 635.376$$